DDU GORAKHPUR UNIVERSITY GORAKHPUR

DEPARTMENT OF MATHEMATICS AND STATISTICS

National Education Policy-2020

Content

Of

Skill Enhancement Course (SEC)

In

Basic Arithmetic

offered by

Department of Mathematics and Statistics

For

UG Programme

Course Title	**Course Code**	**Credits**	**Pre-requisite for Course**	**Elective for SEC**
Basic Arithmetic	SE 1MAT	**3+0**	Mathematics in10^{th}	Open to all

COURSE OBJECTIVES

The course objectives of a Skill Enhancement Course (SEC) in Basic Arithmetic include:

- Developing mathematical thinking: Students may learn to apply math formulas to solve problems.
- Improving analytical and problem-solving skills: Students may learn to analyze and solve problems.
- Improving verbal and communicative skills: Students may learn to improve their verbal and communicative skills.
- Learning to analyze and evaluate accuracy: Students may learn to analyze and evaluate the accuracy of numerical problems.

COURSE OUTCOMES

- The program outcome is to give foundation knowledge for the students to understand basics of mathematics including applied aspect for developing enhanced quantitative skills and pursuing higher study.
- By the time students complete the course; they will have wide ranging application of the subject and have the knowledge of Arithmetic, Reasoning and Logic.
- The main objective of the course is to equip the student with necessary analytic and technical skills. By applying the principles of basic mathematics, he/she learns to solve a variety of practical problems in science, social science, engineering, Commerce and Management etc.

COURSE STRUCTURES

Unit	Topics
	BASIC ARITHMETIC
I	Numbers, H.C.F and L.C.M., Decimal Fraction, Simplification, Square roots and cube roots, Average, Problems of Numbers, Problems on Age.
II	Surds and Indices, Percentage, Profit and Loss, Ratio and Proportion, Partnership, Time and Work, Time and Distance, Problems on Trains, Simple Interest, Compound Interest.
III	Area, Volume and Surface area, Polygons, True Discount, Banker's Discount, Calendar, Clock, Pie Chart, Line Chart and Bar Diagrams.

UNIT-I

Numbers, H.C.F and L.C.M., Decimal Fraction, Simplification, Square roots and cube roots, Average, Problems of Numbers, Problems on Age.

1.1 Numbers:

A **number** is a mathematical object used to count, measure, and label. Numbers can be used for various purposes, including performing arithmetic operations, representing quantities, and solving problems. The concept of a number extends beyond simple counting to more abstract mathematical ideas.

1.2 Types of Numbers:

1. Natural Numbers (N)

- **Definition**: The set of positive integers used for counting and ordering.
- **Examples**: 1, 2, 3, 4, 5…
- **Note**: Natural numbers do not include zero or negative numbers.

2. Whole Numbers (W)

- **Definition**: The set of natural numbers including zero.
- **Examples**: 0, 1, 2, 3, 4, 5, ...
- **Note**: Whole numbers do not include any negative numbers or fractions.

3. Integers (Z)

- **Definition**: The set of all positive and negative whole numbers, including zero.
- **Examples**: -3, -2, -1, 0, 1, 2, 3...
- **Note**: Integers include negative numbers as well as zero and positive numbers.

4. Rational Numbers (Q)

- **Definition**: Numbers that can be expressed as the quotient or fraction of two integers

  (i.e. $\frac{p}{q}$; where p and q both are integers , $q \neq 0$)

- **Examples**: 12,−34,0.75,1.5,0.2,$\frac{3}{4}$, 0.75, $\frac{-25}{7}$.
- **Note**: Rational numbers can be represented as terminating or repeating decimals.

5. Irrational Numbers (I)

- **Definition**: Numbers that cannot be expressed as the quotient of two integers, and their decimal expansions are non-terminating and non-repeating.
- **Examples**: π,$\sqrt{2}$,$\sqrt{3}$.
- **Note**: Irrational numbers cannot be written exactly as fractions.

6. Real Numbers (R)

- **Definition**: The set of all rational and irrational numbers. Essentially, this includes every number that can be represented on the number line.

- **Examples**: -1, 0, 1.5,$\sqrt{7}$,π, 3, -4.25, $\frac{7}{4}$.
- **Note**: Real numbers include both rational and irrational numbers.

9. Prime Numbers

- **Definition**: Natural numbers greater than 1 that have no divisors other than 1 and themselves.
- **Examples**: 2, 3, 5, 7, 11, 13, 17,…
- **Note**: The number 1 is not considered a prime number.

10. Composite Numbers

- **Definition**: Natural numbers greater than 1 that are not prime, meaning they have divisors other than 1 and themselves.
- **Examples**: 4, 6, 8, 9, 10, 12, 14…
- **Note**: Composite numbers can be factored into smaller integers.

11. Even Numbers

- **Definition**: Integers that is divisible by 2.
- **Examples**: -4, -2, 0, 2, 4, 6, 8...
- **Note**: Even numbers can be positive, negative, or zero.

12. Odd Numbers

- **Definition**: Integers that is not divisible by 2.
- **Examples**: -3, -1, 1, 3, 5, 7, 9…
- **Note**: Odd numbers can also be positive or negative.

1.3 HCF (Highest Common Factor):

- **Definition**: The **Highest Common Factor (HCF)**, also known as the **Greatest Common Divisor (GCD)**, of two or more numbers is the largest number that divides each of them exactly without leaving a remainder.

  In other words, HCF is the greatest factor that is common to all the given numbers.

Example 1: Find the HCF of 12 and 18.

Solution:

Step 1: List the factors of each number:

- Factors of 12 = 1, 2, 3, 4, 6, 12
- Factors of 18 = 1, 2, 3, 6, 9, 18

Step 2: Identify the common factors:

- Common factors of 12 and 18: 1, 2, 3, 6

Step 3: The highest common factor is **6**.

So, the **HCF of 12 and 18 is 6**.

Example 2: Find the HCF of 1365, 1560 and 1755.

Solution:

Step 1: List the factors of each number:

- Factors of 1365 = 3, 5, 7, 13
- Factors of 1560 = 2, 2, 2, 3, 5, 13
- Factors of 1755 = 3, 3, 5, 13

Step 2: Identify the common factors:

- Common factors: 3, 5, 13

Step 3: multiply the common factors:

$$\mathbf{3 \times 5 \times 13 = 195}$$

HCF of 1365, 1560 and 1755 is 195.

Example 3: Find the HCF of 180 and 252.

Solution:

Step 1: factorize each number

- Factors of 180 = 2, 2, 3, 3, 5
- Factors of 252 = 2, 2, 3, 3, 7

Step 2: Identify the common factors:

- Common factors: 2, 2, 3,3

Step 3: multiply the common factors:

$$\mathbf{2 \times 2 \times 3 \times 3 = 36}$$

Hence, HCF of 180 and 252 is 36.

1.4 LCM (Lowest Common Multiple):

- **Definition**: The **Lowest Common Multiple (LCM)** of two or more numbers is the smallest number that is a multiple of each of the given numbers.

  In other words, LCM is the smallest positive integer that can be exactly divided by each of the given numbers.

Example 1: Find the LCM of 12 and 18.

Solution:

Step 1: List the multiples of each number:

- Multiples of 12: 12, 24, 36, 48, 60, 72,...
- Multiples of 18: 18, 36, 54, 72, 90, 108,...

Step 2: Identify the common multiples:

- Common multiples of 12 and 18: 36, 72,...

Step 3: The smallest common multiple is **36**.

So, the **LCM of 12 and 18 is 36**.

Example 2: Find the LCM of 45, 72, and 150.
Solution:
Prime factorization of 45: $3 \times 3 \times 5$
Prime factorization of 72: $2 \times 2 \times 2 \times 3 \times 3$
Prime factorization of 150: $2 \times 3 \times 5 \times 5$
Step 1: Identify All Prime Factors
The prime factors involved are 2,3,5
Step 2: Choose the Highest Powers of All Prime Factors
For 2, the highest power is 2^3
For 3, the highest power is 3^2
For 5, the highest power is 5^2
Step 3: LCM (45, 72,150) $=2^3\times3^2\times5^2$
Hence, the LCM of 45, 72, and 150 is $\mathbf{2^3\times3^2\times5^2=1800}$.
Example 3: Find the LCM of 288 and 840.

Solution: Step 1: Factorize of each number:

Prime factorization of 288 $=2^5 \times 3^2$
Prime factorization of 840 $= 2^3 \times 3 \times 5 \times 7$

Step 2: The prime factors involved are 2, 3,5, and 7. We now take the highest powers of each prime factor:

- For 2, the highest power is 2^5.

- For 3, the highest power is 3^2.
- For 5, the highest power is 5^1.
- For 7, the highest power is 7^1.

Step 3: the LCM is the product of the highest powers of all prime factors:

LCM (288,840)=$2^5 \times 3^2 \times 5^1 \times 7^1$= 10080

Hence, the LCM of 288 and 840 is $\mathbf{2^5 \times 3^2 \times 5^1 \times 7^1 = 1800}$.

1.5 Decimal Fraction:

A **decimal fraction** is a type of fraction where the denominator is a power of 10 (i.e., 10, 100, 1000, etc.), and it is written in the form of a decimal number. Decimal fractions are a special case of rational numbers, and they are widely used for representing parts of a whole.

A decimal fraction is a fraction whose denominator is a power of 10. It can also be expressed as a decimal number, where the decimal point separates the whole number part from the fractional part.

Examples of Decimal Fractions:

1. Decimal Examples:

- **0.25** = 1/4
- **1.75** = 7/4
- **0.125** = 1/8

2. Use of Equation for Fractions:

- 1/4 = 0.25
- 7/4 = 1.75
- 1/8 = 0.125

NOTE:

- The **denominator** is always a power of 10 (10, 100, 1000, etc.).
- Decimal fractions can be **terminating** (end after a few digits) or **repeating** (have a repeating pattern after the decimal point).
- They are **rational numbers**, meaning they can be expressed as fractions.

1.6 Simplification:

Simplification refers to the process of reducing an expression or equation to its simplest or most basic form, making it easier to understand or solve. This involves performing mathematical operations such as addition, subtraction, multiplication, or division to combine terms or remove unnecessary components.

In general, simplification can apply to **expressions**, **fractions**, **equations**, or **algebraic terms**.

1.7 Key Concepts in Simplification:

1. **Simplifying Fractions:** The process of reducing a fraction to its lowest terms by dividing both the numerator and the denominator by their greatest common divisor (GCD).

Example: Simplifying $\frac{12}{16}$

Step 1: Find the GCD of 12 and 16

- The **factors of 12** are: 1, 2, 3, 4, 6, 12
- The **factors of 16** are: 1, 2, 4, 8, 16

The **GCD** of 12 and 16 is **4** because 4 is the largest number that divides both 12 and 16.

Step 2: Divide both the numerator and the denominator by the GCD

- **Numerator**: $12 \div 4 = 3$
- **Denominator**: $16 \div 4 = 4$

So, $\frac{12}{16}$ simplifies to $\frac{3}{4}$

2. **Simplifying Algebraic Expressions:** Combining like terms or removing parentheses to reduce the complexity of the expression.

Example: Simplifying an Expression

Expression: $5x + 3x - 7 + 2$

Solution: Step 1: Combine like terms

- The like terms are the terms with the same variable, in this case, the terms with **x**. Combine 5x and3x:

$$5x+3x=8x$$

- Combine the constants −7and 2:

$$-7+2=-5$$

Step 2: Write the Simplified Expression

Now, write the simplified expression:

$$8x-5$$

Simplified Expression**:**

$$8x-5$$

3. **Simplifying Square Roots:** Expressing square roots or other radical expressions in their simplest form by factoring out perfect squares.

- **Steps for Simplifying Square Roots:**

1. **Factor the number under the square root** into its prime factors.
2. **Identify perfect square factors** and take their square root.
3. **Simplify the square root** by bringing out the square root of the perfect squares.
4. **Write the final simplified expression.**

Example: Simplifying $\sqrt{72}$

Solution: The prime factorization of 72 is:

$$72=2\times2\times2\times3\times3$$

Identify perfect square factors

From the factorization, we see that $2\times2=4$and $3\times3=9$, both of which are perfect squares.

Step 3: Simplify the square root

We can rewrite$\boldsymbol{\sqrt{72}}$ as:

$$\boldsymbol{\sqrt{72} = \sqrt{4\times9\times2}}$$

Now, simplify by taking the square root of the perfect squares (4 and 9):

$$\boldsymbol{\sqrt{72} = \sqrt{4}\times\sqrt{9}\times\sqrt{2} = 2\times3\times\sqrt{2} = 6\sqrt{2}}$$

Simplified Expression**:**

$$\boldsymbol{\sqrt{72} = 6\sqrt{2}}$$

4. **Simplifying Equations:** Solving or reducing an equation to a simpler form for easier interpretation or solution.

Example: Simplify and Solve the Equation 4(x+5) = 28

Solution: Given equation is

$$4(x+5) =28$$

Step 1: Distribute the 4 to both terms inside the parentheses

We start by using the **distributive property** to simplify the equation. Multiply the 4 by both x and 5:

$$4\times x+4\times5=4x+20$$

So, the equation becomes:

$$4x+20=28$$

Step 2: Subtract 20 from both sides

To isolate the term with x, subtract 20 from both sides of the equation:

$$4x+20-20=28-20$$

This simplifies to:

$$4x=8$$

Step 3: Divide both sides by 4

Now, to solve for x, divide both sides of the equation by 4:

$$\frac{4x}{4}=\frac{8}{4}$$

Simplifying both sides:

$$x = 2$$

Hence, the solution to the equation is:

$$x = 2$$

1.8 Square Root(√):

- The square root of a number is a value that, when multiplied by itself, gives the original number.
- Mathematically, the square root of x is denoted as $\sqrt{x}$, and it is the number y such that $y^2=x$

Example: Find the square root of 625.

Solution: The square root of 625 is the number that, when multiplied by itself, equals 625.

$$\sqrt{\mathbf{625}} = 25 \ \text{(because } 25 \times 25 = 625\text{)}$$

Similarly, for negative numbers:

$$\sqrt{\mathbf{169}} = 13 \ \text{(since } 13 \times 13 = 169\text{)}$$

In this case, both 14 and −14 are square roots of 196 because:

$$\mathbf{14^2} = 196 \ \text{and} \ (-14)^2 = 196$$

Thus, the square root of a positive number has two solutions: one positive and one negative.

1.9 Cube Root ($\sqrt[3]{}$):

- The cube root of a number is a value that, when multiplied by itself three times, gives the original number.
- Mathematically, the cube root of x is denoted as $\sqrt[3]{x}$ and it is the number y such that $y^3 = x$

In other words, $\sqrt[3]{x}$= y if and only if $y^3 = x$.

Example: Find the cube root of 512.

The cube root of 512 is the number that, when multiplied by itself three times, equals 512.

$$\sqrt[3]{512} = 8 \text{ because } 8 \times 8 \times 8 = 512$$

Similarly, for negative numbers:

$$\sqrt[3]{-8} = -2 \text{ (because } (-2) \times (-2) \times (-2) = -8)$$

In this case, unlike square roots, the cube root of a negative number is also negative.

1.10 Mean (Average):

The **mean**, commonly referred to as the **average**, is calculated by adding up all the values in a data set and then dividing by the number of values in that set.

Mathematically, the **mean** is calculated as:

$$\text{Mean} = \frac{\text{Sum of all values}}{\text{number of values}}$$

Example: Find average of the following five test scores: 80, 90, 100, 85, and 95.

Solution:

1. **Step 1: Add all the values together.**

   80+90+100+85+95=450

2. **Step 2: Count how many values there are.**

   There are 5 test scores.

3. **Step 3: Divide the sum by the number of values.**

$$\text{Mean} = \frac{450}{5} = 90$$

Hence, **average** of the five test scores is **90**

1.11 Median:

The **median** is the middle value in a set of numbers when the numbers are arranged in ascending or descending order.

- If there is an **odd** number of values, the median is the middle number.
- If there is an **even** number of values, the median is the average of the two middle numbers.

Example 1: Median for an Odd Set of Numbers

Solution: For the numbers: **80, 85, 90, 95, 100**

1. Arrange the numbers in order (if not already): **80, 85, 90, 95, 100**.
2. Since there are 5 numbers (an odd number), the median is the middle number: **90**.

So, the **median** is**90.**

Example 2: Median for an Even Set of Numbers

Solution: For the numbers: **80, 85, 90, 95, 100, 105**

1. Arrange the numbers in order: **80, 85, 90, 95, 100, 105**.
2. Since there are 6 numbers (an even number), we find the two middle numbers: **90** and **95**.
3. To find the median, calculate the average of these two middle numbers:

$$\text{Median} = \frac{90 + 95}{2} = \frac{185}{2} = 92.5$$

So, the **median** is**92.5.**

1.12 Mode:

The **mode** is the value that appears most frequently in a set of numbers.

- A set of numbers can have **one mode**, **more than one mode**, or **no mode** (if all values appear only once).

Example 1: Mode for a Set of Numbers

Solution: For the numbers: **80, 90, 90, 100, 100**

1. Count how many times each number appears:
   - 80 appears once.
   - 90 appears twice.

   - 100 appears twice.
2. The numbers that appear most frequently are **90** and **100** (each appears twice).

So, the **mode** is:
90 and 100 (this is called a **bimodal** set).

Example 2: Mode for a Set with One Mode

Solution: For the numbers: **70, 80, 85, 90, 100**

1. Count how many times each number appears:
   - 70 appears once.
   - 80 appears once.
   - 85 appears once
   - 90 appears once.
   - 100 appears once.

Since no number repeats, the set has **no mode**.

So, the **mode** is**none.**

1.13 Problems of Numbers:

1. **Problems Involving Averages:**

- **Mean Problem:**

Problem: The ages of five students are 15, 16, 14, 18, and 17. What is the average age?

Solution:

$$\text{Average(mean)} = \frac{15 + 16 + 14 + 18 + 17}{5} = \frac{80}{5} = 16$$

So, the **average age** of the students is **16**.

- **Median Problem:**

Problem: The heights (in cm) of five students are 160, 155, 170, 165, and 160. What is the median height?

Solution: First, arrange the heights in order:

155, 160, 160, 165, 170.

The **median** is the middle value: 160.

2. **Number Theory Problems:**

- **Prime Numbers:**

Problem: Is 29 a prime number?

Solution: Yes, 29 is a prime number because it has no divisors other than 1 and 29.

- **Divisibility Rules:**

Problem: Is 123 divisible by 3?

Solution: Yes, because the sum of the digits (1 + 2 + 3 = 6) is divisible by 3.

Hence, the 123 is divisible by 3.

3. **Word Problems:**

- **Percentage Problem:**

Problem: A jacket costs Rs 80, and it's on sale for 25% off. What is the sale price of the jacket?

Solution: To calculate the discount, we find 25% of the original price (Rs 80). To do this,

multiply the original price by 25% (or 0.25):

$$\text{Discount} = 80 \times 0.25 = 20$$

So, the discount amount is **Rs 20**

Now, find the sale price, subtract the discount from the original price:

$$\text{Sale Price} = 80 - 20 = 60$$

Hence, the **sale price** of the jacket is **Rs 60**.

4. **Algebraic Problems:**

- **Solving for x:**

Problem: Solve for x:

$$2x+5=15$$

Solution: Given equation is

$$2x+5=15$$

Subtract 5 from both sides, we have

$$2x=10$$

Divide by 2, we have

$$x=5$$

5. **Advanced Number Problems:**

- **Pythagorean Theorem (Right Triangles):**

Problem: In a right triangle, the lengths of the two legs are 3 and 4 units. What is the length of the hypotenuse?

Solution: The **Pythagorean Theorem** states that in a right triangle,

the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the two legs a and b.

Mathematically, the theorem is expressed as:

- Use the Pythagorean theorem:

$$c^2 = a^2 + b^2$$

Where:

- c is the length of the hypotenuse,
- a and b are the lengths of the two legs.

Substitute the known values

In this problem, the lengths of the two legs are:

- $a = 3$
- $b = 4$

Now, substitute these values into the Pythagorean Theorem:

$$c^2 = 3^2 + 4^2$$

$$c^2 = 9 + 16$$

$$c^2 = 25$$

Solve for c:

To find the length of the hypotenuse, take the square root of both sides:

$$c = \sqrt{25}$$

$$c = 5$$

- The hypotenuse is **5 units**.

1.14 Age-Related Problems (Word Problems):

Age problems are a common type of algebraic word problem. They often involve relationships between the ages of two or more people, either in the present or in the past/future. Below are various types of age-related problems with their explanations and examples:

1. **Basic Age Difference Problems:**

Problem Type:

Two people have a certain age difference, and you are asked to find their ages based on this difference.

Example 1: John is 5 years older than Mary. If Mary is 12 years old, how old is John?

Solution:

- We are told that John is 5 years older than Mary.
- Mary's age = 12 years.
- John's age = Mary's age + 5

= 12+5

=17.

So, John is 17 years old.

2. **Age in the Past or Future:**

Problem Type:

The problem asks you to find someone's age in the past or future, based on their age today and the relationship between the ages at different times.

Example 2: Ten years ago, Lisa was three times as old as Tom. Now, Lisa is 25 years old. How old is Tom now?

Solution:

Let Tom's current age be x.

Ten years ago, Lisa's age was

$$25-10=15.$$

Ten years ago, Tom's age was

$$x-10.$$

We are told that, ten years ago, Lisa was three times as old as Tom. So, we set up the equation:

$$15=3(x-10)$$

Solve for x:

$\Rightarrow 15=3x-30$

$\Rightarrow 45=3x$

$\Rightarrow 3x = 45$

$\Rightarrow x = \frac{45}{3}$

$\Rightarrow x = 15$

Hence, Tom is currently **15 years old**.

3. Comparing Ages (with Future or Past Relationships):

Problem Type:

One person's age is a multiple or fraction of another person's age, either now or in the past/future.

Example 3: **Five years ago, Jane was twice as old as Peter. Now, Jane is 30 years old. How old is Peter now?**

Solution:

Let Peter's current age be x.

Five years ago,

Jane's age was $30-5=25$

Five years ago,

Peter's age was $x-5$

We are told that five years ago, Jane was twice as old as Peter.

So, we set up the equation:

$$25=2(x-5)$$

Solve for x:

$\Rightarrow 25=2x-10$

$\Rightarrow 35=2x$

$\Rightarrow$ x =35

$\Rightarrow x = \frac{35}{2}$

$\Rightarrow x = 17.5$

Hence, Peter is currently **17.5 years old** (or 17 years and 6 months).

4. **Age Ratio Problems:**

Problem Type:

The ages of two or more people are related by a ratio.

Example 4: The ratio of the ages of Alice and Sam is 4:3. If Alice is 24 years old, how old is Sam?

Solution: Let the ages of Alice and Sam be represented as multiples of the ratio

Let us suppose Alice's age be 4x and Sam's age be 3x.

Use the given information about Alice's age

We are told that Alice is 24 years old.

So, we can set Alice's age equal to 4x,

$$4x = 24$$

Solve for x:

To find x, divide both sides of the equation by 4:

$x = \frac{24}{4}$

$\Rightarrow x = 6$

Find Sam's age

Now that we know x = 6,

we can find Sam's age by multiplying 3 by x:

Sam's age = 3x

$= 3 \times 6$

$= 18$

Hence, Sam is **18 years old**.

5. **Sum of Ages Problems:**

Problem Type:

The sum of two or more people's ages is given, and you are asked to find their individual ages.

Example 5: The sum of the ages of Anna and her brother is 50 years. Anna is 6 years older than her brother. How old is each of them?

Solution: Define the variables

Let us suppose that A be Anna's age and B be her brother's age.

From the problem, we have two key pieces of information:

The sum of their ages is 50 years:

$$A + B = 50$$

And Anna is 6 years older than her brother:

$$A = B + 6$$

We have two equations:

$A+B=50$ ... (1)

and $A=B+6$ … (2)

Now, substitute the expression for A from the second equation into the first equation:

$$(B + 6) + B = 50$$

Solve for B

Simplify the equation:

$2B + 6 = 50$

Now, subtract 6 from both sides:

$2B = 44$

Next, divide both sides by 2:

$B=22$

Find Anna's age

Now that we know $B = 22$ (her brother's age), we can find Anna's age by substituting B into the equation $A = B + 6$

$A = 22 + 6$

$= 28$

Hence, Anna is 28 years old and Her brother is 22 years old.

6. **Age Difference in the Future:**

Problem Type:

You are asked to find the ages of two people at some point in the future, given their current ages and **the time in the future.**

Example 6*:* In 10 years, Peter will be 3 times as old as his sister Lisa. If Peter is currently 20 years old, how old is Lisa now?

Solution: Let Peter's current age be P and Lisa's current age be L.

We know that Peter's current age is 20 years, so:

$P = 20$

In 10 years,

Peter will be 3 times as old as Lisa.

In10 years, Peter's age will be P + 10 and Lisa's age will be L+10.

The problem tells us that in 10 years,

Peter will be 3 times as old as Lisa:

$P + 10 = 3(L + 10)$

Substitute Peter's age:

We know that P = 20, so substitute this into the equation:

$20 + 10 = 3(L+10)$

Solve for Lisa's age:

Now, simplify the equation:

$30 = 3(L+10)$

First, divide both sides by 3:

$10 = L+10$

Now, subtract 10 from both sides:

$L = 0$

Hence, Lisa is currently 0 years old. This means she is just born.

Example 7: The difference between the ages of a mother and her daughter is 24 years. Four years ago, the mother was 4 times as old as her daughter. What are their present ages?

Solution:

Let the present age of the mother be M and the present age of the daughter be D.

Step 1: Translate the problem into equations

- **First equation** (difference in their ages):
  The difference between the mother's age and the daughter's age is 24 years:

$$M – D=24$$

- **Second equation** (relationship four years ago):
  Four years ago, the mother's age was M−4and the daughter's age was D−4.
  According to the problem, four years ago, the mother was 4 times as old as her daughter:

$$M - 4 = 4(D - 4)$$

Step 2: Solve the system of equations

1. From the first equation:

$$M - D = 24$$

$$\Rightarrow M = D + 24$$

2. Substitute $M=D+24$ into the second equation:

$$(D+24) - 4 = 4(D-4)$$

3. Simplify:

$$D+20 = 4(D-4)$$

4. Rearrange the equation:

$$D - 4D = -16 - 20$$

5. Solve for D:

$$-3D = -36$$

$$\Rightarrow D = 12$$

Step 3: Find M

Now substitute $D=12$ into the first equation $M=D+24$:

$$M = 12 + 24 = 36$$

Hence, the **mother's present age** is **36 years**, and the **daughter's present age** is **12 years**.

UNIT-II

Surds and Indices, Percentage, Profit and Loss, Ratio and Proportion, Partnership, Time and Work, Time and Distance, Problems on Trains, Simple Interest, Compound Interest.

2.1 Surds:

A surd is an expression involving roots that cannot be simplified into a rational number. Typically, it refers to square roots (or other roots) of numbers that are **irrational**.

2.2 Definition of a Surd:

A surd is an expression of the form ($\sqrt{a}$)where (a)is a positive number and (a)is **not aperfect square** (i.e., ($\sqrt{a}$)is irrational). Surds can involve higher roots, such as cube roots, fourth roots, etc.

For example:

- ($\sqrt{2}$)is a surd because 2 is not a perfect square, so ($\sqrt{2}$)is irrational.
- ($\sqrt{4} = 2$)is not a surd because 4 is a perfect square and the square root of 4 is a rational number (2).

2.3 Simplifying Surds:

You can simplify surds by factoring the number under the root and pulling out perfect squares (or cubes, etc.):

For example:

- ($\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$).

- ($\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$).

You cannot simplify surds to a rational number unless the radicand (the number inside the root) is a perfect square (for square roots), perfect cube (for cube roots), etc.

2.4 Rationalizing the Denominator:

If a surd appears in the denominator of a fraction, you may need to rationalize it by multiplying both the numerator and denominator by an appropriate surd to eliminate the surd in the denominator.

For example:

To rationalize ($\frac{1}{\sqrt{2}}$),multiply both the numerator and the denominator by ($\sqrt{2}$):

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

2.5 Indices (Exponents):

Indices (or exponents) are a shorthand notation used to express repeated multiplication of the same number. They indicate how many times a number (called the base) is multiplied by itself.

2.6 Basic Rules of Indices:

1. Product Rule:[$a^m \times a^n = a^{m+n}$] When multiplying powers with the same base, add the exponents.

2. Quotient Rule:$[\frac{a^m}{a^n} = a^{m-n}]$ When dividing powers with the same base, subtract the exponents.

3. Power Rule: $[(a^m)^n = a^{m\times n}]$ When raising a power to another power, multiply the exponents.

4. Zero Exponent Rule:$[a^0 = 1]$ Any non-zero number raised to the power of zero is 1.

5. Negative Exponent Rule: $[a^{-n} = \frac{1}{a^n}]$ A negative exponent represents the reciprocal of the number with the positive exponent.

6. Fractional Exponent Rule: $[a^{\frac{m}{n}} = \sqrt[n]{a^m}]$ A fractional exponent represents the root of the number. For example, $(a^{\frac{1}{2}})$is the square root of (a), and $(a^{\frac{1}{3}})$is the cube root of (a).

Example of Applying Indices:

- $(2^3 \times 2^4 = 2^{3+4} = 2^7 = 128)$
- $(\frac{5^6}{5^2} = 5^{6-2} = 5^4 = 625)$
- $((3^2)^3 = 3^{2\times 3} = 3^6 = 729)$
- $(4^0 = 1)$
- $(2^{-3} = \frac{1}{2^3} = \frac{1}{8})$
- $(16^{\frac{1}{4}} = \sqrt[4]{16} = 2)$

2.7 Relationship between Surds and Indices:

Surds and indices are closely related, particularly when working with roots and fractional exponents:

- **Square Roots as Exponents:** The square root of a number can be written as an exponent:

$$[\sqrt{a} = a^{\frac{1}{2}}]$$

For example, $(\sqrt{9} = 9^{\frac{1}{2}} = 3)$.

- **Higher Roots and Fractional Exponents:** Cube roots, fourth roots, etc., can also be expressed as fractional exponents:

$[\sqrt[3]{a} = a^{\frac{1}{3}}, \quad \sqrt[4]{a} = a^{\frac{1}{4}}]$ For instance, $(\sqrt[3]{27} = 27^{\frac{1}{3}} = 3)$.

- **Combining Surds and Indices:** You can simplify expressions involving both surds and indices:

$$[\sqrt{a} \times \sqrt{b} = a^{\frac{1}{2}} \times b^{\frac{1}{2}} = (ab)^{\frac{1}{2}}]$$

For example: $[\sqrt{2} \times \sqrt{3} = (2 \times 3)^{\frac{1}{2}} = \sqrt{6}]$

Some Examples

Example 1: Simplify $(\sqrt{50})$.

Solution: $[\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}]$So, the simplified form of $(\sqrt{50})$ is $(5\sqrt{2})$.

Example 2: Simplify $(\sqrt{12} \times \sqrt{3})$.

Solution:

Using the property $(\sqrt{a} \times \sqrt{b} = \sqrt{ab})$,we get:

$[\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = 6]$So, $(\sqrt{12} \times \sqrt{3} = 6)$.

Example 3: Simplify $((2\sqrt{3})^2)$.

Solution: First, square both the coefficient and the surd:

$[(2\sqrt{3})^2 = 2^2 \times (\sqrt{3})^2 = 4 \times 3 = 12]$So, $((2\sqrt{3})^2 = 12)$.

Example 4: Simplify $(\frac{2\sqrt{5}}{3\sqrt{7}})$.

Solution: First, simplify the fraction by factoring out the square roots:

$[\frac{2\sqrt{5}}{3\sqrt{7}} = \frac{2}{3} \times \frac{\sqrt{5}}{\sqrt{7}} = \frac{2}{3} \times \sqrt{\frac{5}{7}}]$So, the simplified form is$[\frac{2}{3} \times \sqrt{\frac{5}{7}} = \frac{2\sqrt{5}}{3\sqrt{7}}]$

Example 5: Simplify $(\frac{2^5}{2^2} \times 2^3)$.

Solution: $(\frac{a^m}{a^n} = a^{m-n})$

$[\frac{2^5}{2^2} = 2^{5-2} = 2^3]$

Now multiply by (2^3): $[2^3 \times 2^3 = 2^{3+3} = 2^6]$

So, the simplified result is $(2^6 = 64)$.

2.8 Percentages:

A percentage is simply a way of expressing a number as a fraction of 100. The term comes from the Latin word "per centum," which means "by the hundred." It is denoted by the symbol `%` and is used to express how much of something is in relation to a whole.

2.9 Basic Formula for Percentage:

The general formula for calculating the percentage of a number is:

$$[\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100]$$

Where:

Part is the portion or amount of the whole you're interested in.

Whole is the total or the full amount that the part is being compared to.

Multiply by 100 to convert the fraction to a percentage.

2.10 Basic Operations with Percentages:

1. Finding the Percentage of a Number

To find a percentage of a given number, multiply the number by the percentage (expressed as a decimal).

$$[\text{Percentage of a Number} = \frac{\text{Percentage}}{100} \times \text{Number}]$$

Example 1: Find 25% of 80.

$$[\frac{25}{100} \times 80 = 0.25 \times 80 = 20]$$

So, 25% of 80 is 20.

2. Finding the Whole When a Percentage is Given

If you know the part and the percentage, you can find the whole by rearranging the percentage formula:

$$[\text{Whole} = \frac{\text{Part}}{\text{Percentage}} \times 100]$$

Example 2: If 30% of a number is 60, what is the number?

$$[\text{Whole} = \frac{60}{30} \times 100 = 2 \times 100 = 200]$$

So, the whole is 200.

3. Finding the Percentage Change

The percentage change is calculated using the formula:

$$[\text{Percentage Change} = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100]$$

Example 3: A shirt originally cost Rs.50 but is now on sale for Rs.40. What is the percentage decrease in the price?

$$[\text{Percentage Decrease} = \frac{50 - 40}{50} \times 100 = \frac{10}{50} \times 100 = 20\%]$$

So, the price has decreased by 20%.

4. Increasing a Number by a Percentage

If you need to increase a number by a given percentage, you can multiply the number by

$$(1 + \frac{\text{Percentage}}{100}).$$

Example 4: A salary of Rs.2,000 is increased by 10%. What is the new salary?

$$[\text{New Salary} = 2000 \times \left(1 + \frac{10}{100}\right) = 2000 \times 1.10 = 2200]$$

So, the new salary is Rs.2,200.

5. Decreasing a Number by a Percentage

To decrease a number by a given percentage, multiply the number by

$$(1 - \frac{\text{Percentage}}{100}).$$

Example 5: A product priced at Rs.150 is discounted by 20%. What is the new price?

$$[\text{New Price} = 150 \times \left(1 - \frac{20}{100}\right) = 150 \times 0.80 = 120]$$

So, the new price after the discount is Rs.120.

Examples of Percentage Calculations

1. Converting a Percentage to a Fraction or Decimal

Example 6: Convert 40% to a decimal.

$$[40\% = \frac{40}{100} = 0.40]$$

So, 40% is 0.40 in decimal form.

Example 7: Convert 5% to a fraction.

$$5\% = \frac{5}{100} = \frac{1}{20}$$

So, 5% is $(\frac{1}{20})$ in fractional form.

2. Calculating Percentage Increase/Decrease

Example 8: The population of a town increases from 10,000 to 12,500. What is the percentage increase?

$$[\text{Percentage Increase} = \frac{12{,}500 - 10{,}000}{10{,}000} \times 100 = \frac{2{,}500}{10{,}000} \times 100 = 25\%]$$

So, the population has increased by 25%.

3. Finding the Percentage of an Amount

Example 9: What is 15% of Rs.600?

$$\frac{15}{100} \times 600 = 0.15 \times 600 = 90$$

So, 15% of Rs.600 is Rs.90.

4. Finding the Total Amount after a Discount

Example 10: A jacket costs Rs.120, and there is a 25% discount. How much will you pay for the jacket?

$$[\text{Discount} = \frac{25}{100} \times 120 = 30]$$

So, the discount is Rs.30. The price after the discount is:

$$[120 - 30 = 90]$$

So, the jacket will cost Rs.90 after the discount.

Some Examples:

Examples 1: A company offers its employees a 15% salary increase. If John's current salary is Rs.40, 000 per year, how much will his salary increase by, and what will his new salary be?

Solution:

1. Calculate the salary increase:

The salary increase is 15% of John's current salary.

$$[\text{Salary Increase} = \frac{15}{100} \times 40{,}000 = 0.15 \times 40{,}000 = 6{,}000]$$

So, John will receive a salary increase of Rs.6,000.

2. Calculate the new salary:

John's new salary will be his current salary plus the increase:

$$[\text{New Salary} = 40{,}000 + 6{,}000 = 46{,}000]$$

So, John's new salary will be Rs.46,000.

Example 2: A jacket is originally priced at Rs.120, but there is a 25% discount on the jacket. How much will you pay for the jacket after the discount?

Solution:

1. Calculate the discount:

The discount on the jacket is 25% of Rs.120:

$$[\text{Discount} = \frac{25}{100} \times 120 = 0.25 \times 120 = 30]$$

So, the discount is Rs.30.

2. Calculate the price after the discount:

The price after the discount is the original price minus the discount:

$$[\text{Price After Discount} = 120 - 30 = 90]$$

So, the price you will pay after the discount is Rs.90.

Example 3: You take out a loan of Rs.5,000 with an annual interest rate of 8%. How much interest will you pay after 1 year?

Solution:

The interest is 8% of the principal loan amount (Rs.5,000).

$$[\text{Interest} = \frac{8}{100} \times 5{,}000 = 0.08 \times 5{,}000 = 400]$$

So, the interest you will pay after 1 year is Rs.400.

Example 4: The population of a town is 50,000. It grows at a rate of 3% per year. What will be the population after 1 year?

Solution:

1. Calculate the increase in population:

The increase in population is 3% of 50,000.

$$[\text{Population Increase} = \frac{3}{100} \times 50{,}000 = 0.03 \times 50{,}000 = 1{,}500]$$

So, the population will increase by 1,500.

2. Calculate the new population:

The new population after 1 year will be the current population plus the increase:

$$[\text{New Population} = 50{,}000 + 1{,}500 = 51{,}500]$$

So, the population after 1 year will be 51,500.

Example 5: A customer buys a smartphone for Rs.800, and the sales tax rate is 7%. What is the amount of tax, and what will be the total price after tax?

Solution:

1. Calculate the tax:

The tax is 7% of Rs.800:

$$[\text{Tax} = \frac{7}{100} \times 800 = 0.07 \times 800 = 56]$$

So, the sales tax is Rs.56.

2. Calculate the total price after tax:

The total price is the original price plus the tax:

$[\text{Total Price} = 800 + 56 = 856]$So, the total price after tax is Rs.856.

Example 6: A school has 800 students. 40% of the students are enrolled in the school's sports program, and 30% of the students are enrolled in the music program.

1. How many students are enrolled in the sports program?

2. How many students are enrolled in the music program?

3. How many students are enrolled in both the sports and music programs if 10% of the total student population is enrolled in both?

4. How many students are enrolled in either the sports program or the music program (or both)?

Solution:

1. Students in the sports program:

40% of 800 students are in the sports program. To find 40% of 800:

$$[\text{Students in sports program} = \frac{40}{100} \times 800 = 320 \text{ students}]$$

2. Students in the music program:

30% of 800 students are in the music program. To find 30% of 800:

$$[\text{Students in music program} = \frac{30}{100} \times 800 = 240 \text{ students}]$$

3. Students in both the sports and music programs:

10% of 800 students are enrolled in both programs. To find 10% of 800:

$$[\text{Students in both programs} = \frac{10}{100} \times 800 = 80 \text{ students}]$$

4. Students in either the sports program or the music program (or both):

To find the number of students in either program, use the formula for the union of two sets:

$[\text{Students in either program} = (\text{Students in sports}) + (\text{Students in music}) - (\text{Students in both})]$
Substituting the values:

$$[\text{Students in either program} = 320 + 240 - 80 = 480 \text{ students}]$$

Answers:

1. 320 students are enrolled in the sports program.

2. 240 students are enrolled in the music program.

3. 80 students are enrolled in both the sports and music programs.

4. 480 students are enrolled in either the sports program, the music program, or both.

2.11 Profit and Loss:

In business and finance, profit refers to the positive difference between the revenue from sales and the costs of production, while loss refers to the situation where expenses exceed revenue. Profit and loss calculations are essential for determining how well a business is performing.

- **Key Terms:**

1. **Cost Price (C.P.):** The price at which an item is bought.

2. **Selling Price (S.P.):** The price at which an item is sold.

3. **Profit:** The amount gained when the selling price exceeds the cost price.

4. **Loss:** The amount lost when the selling price is less than the cost price.

5. **Profit Percentage:** The profit expressed as a percentage of the cost price.

6. **Loss Percentage:** The loss expressed as a percentage of the cost price.

- **Formulas:**

Profit = Selling Price (S.P.) - Cost Price (C.P.)

Loss = Cost Price (C.P.) - Selling Price (S.P.)

Profit Percentage = $(\frac{\text{Profit}}{\text{Cost Price}} \times 100)$

Loss Percentage = $(\frac{\text{Loss}}{\text{Cost Price}} \times 100)$

Examples of Profit and Loss Problems:

Example 1: A shopkeeper buys a television for Rs.800 and sells it for Rs.1,000. What is his profit?

Solution:

Cost Price (C.P.)= Rs.800

Selling Price (S.P.) = Rs.1,000

Profit = S.P. - C.P. = Rs.1,000 - Rs.800 = Rs.200

So, the shopkeeper makes a profit of Rs.200.

Example 2: A person bought a watch for Rs.500 and sold it for Rs.650. What is the profit percentage?

Solution:

Cost Price (C.P.) = Rs.500

Selling Price (S.P.) = Rs.650

Profit = S.P. - C.P. = Rs.650 - Rs.500 = Rs.150

Now, to calculate the profit percentage:

$$[\text{Profit Percentage} = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{150}{500} \times 100 = 30\%]$$

So, the profit percentage is 30%.

Example 3: A person buys a bag for Rs.120 and sells it for Rs.100. What is the loss?

Solution:

Cost Price (C.P.) = Rs.120

Selling Price (S.P.) = Rs.100

Loss = C.P. - S.P. = Rs.120 - Rs.100 = Rs.20

So, the person incurs a loss of Rs.20.

Example 4: A businessman bought a piece of equipment for Rs.2,000 and sold it for Rs.1,800. What is the loss percentage?

Solution:

Cost Price (C.P.) = Rs.2,000

Selling Price (S.P.) = Rs.1,800

Loss = C.P. - S.P. = Rs.2,000 - Rs.1,800 = Rs.200

Now, to calculate the loss percentage:

$$\text{Loss Percentage} = \frac{\text{Loss}}{\text{C.P.}} \times 100 = \frac{200}{2{,}000} \times 100 = 10\%$$

So, the loss percentage is 10%.

Example 5: A merchant buys 50 shirts for Rs.25 each. He then sells them for Rs.30 each. What is his total profit?

Solution:

Cost Price per Shirt = Rs.25

Selling Price per Shirt = Rs.30

Profit per Shirt = Selling Price - Cost Price = Rs.30 - Rs.25 = Rs.5

Now, for 50 shirts:

Total Profit = Profit per Shirt × Number of Shirts = Rs.5 × 50 = Rs.250So, the total profit is Rs.250.

Example 6: A store buys a laptop for Rs.600 and offers a 10% discount on the selling price. If the selling price before discount was Rs.700, what is the store's profit?

Solution:

Cost Price (C.P.) = Rs.600

Selling Price Before Discount (S.P.) = Rs.700

Discount = 10% of Rs.700 = $(\frac{10}{100} \times 700 = 70)$

So, the selling price after the discount is:

$$[\text{Selling Price After Discount} = 700 - 70 = 630]$$

Now, the profit is:

$$[\text{Profit} = \text{S.P. After Discount} - \text{C.P.} = 630 - 600 = 30]$$

So, the profit is Rs.30.

Example 7: A wholesaler buys a product for Rs.80 and marks it up by 25%. What is the selling price, and what is the profit?

Solution:

1. Markup Calculation:

Markup is 25% of the cost price:

$$[\text{Markup} = \frac{25}{100} \times 80 = 0.25 \times 80 = 20]$$

2. Selling Price:

The selling price is the cost price plus the markup:

$$[\text{Selling Price} = 80 + 20 = 100]$$

3. Profit:

The profit is the markup:$[\text{Profit} = 20]$

So, the selling price is Rs.100, and the profit is Rs.20.

Example 8: A retailer buys 100 books for Rs.12 each. He sells them at Rs.10 each. What is his total loss?

Solution:

Cost Price per Book = Rs.12

Selling Price per Book = Rs.10

Loss per Book = C.P. - S.P. = Rs.12 - Rs.10 = Rs.2

Now, for 100 books:

Total Loss = Loss per Book × Number of Books = Rs.2 × 100 = Rs.200

So, the total loss is Rs.200.

Example 9: Sarah runs a small bakery. She buys ingredients to make cakes and sells them at a local market. The cost to bake one cake is Rs.12, including ingredients, labour, and overhead costs. Sarah sells each cake for Rs.18.

1. How much profit does Sarah make on each cake?

2. If Sarah sells 50 cakes in a day, how much total profit does she make that day?

3. If she sells 60 cakes in a day, how much does her total profit increase compared to selling 50 cakes?

Solution:

1. Profit per cake:

The profit per cake is the selling price minus the cost price:

$$[\text{Profit per cake} = \text{Selling Price} - \text{Cost Price} = 18 - 12 = \text{Rs}.6\ ]$$

2. Total profit for 50 cakes:

If Sarah sells 50 cakes, the total profit is:

$$[\text{Total Profit} = 50 \times 6 = \text{Rs}.300]$$

3. Total profit for 60 cakes:

If Sarah sells 60 cakes, the total profit is:

$$[\text{Total Profit} = 60 \times 6 = \text{Rs}.360\ ]$$

4. Increase in profit from 50 to 60 cakes:

The increase in profit is:

$$[\text{Increase in Profit} = 360 - 300 = \text{Rs}.60\ ]$$

So, the answers are:

1. Sarah makes Rs.6 profit on each cake.

2. Sarah makes Rs.300 total profit for 50 cakes.

3. Her total profit increases by Rs.60 when she sells 60 cakes instead of 50.

2.12 Ratio and Proportion:

Ratio and proportion are fundamental concepts in mathematics that deal with the relationship between quantities. They are used to compare two or more quantities and express their relationship in a defined manner. These concepts are widely used in various real-world applications such as cooking, finance, and measurement.

2.13 Ratio:

A ratio is a way to compare two or more quantities. It tells you how many times one quantity is contained in another. Ratios can be expressed in different ways: as a fraction, with a colon, or using the word "to."

2.14 Types of Ratios:

- **Simple Ratio:** A ratio between two numbers is written as $(a:b)$or$(\frac{a}{b})$, where $(a)$and $(b)$are the two quantities being compared.

Example: If there are 2 boys and 3 girls in a class, the ratio of boys to girls is:

$$[\text{Ratio of boys to girls} = 2:3]$$

- **Expanded Ratio:** A ratio involving more than two quantities.

Example: The ratio of boys, girls, and teachers in a school can be expressed as:

$$[\text{Ratio of boys to girls to teachers} = 2:3:1]$$

- **Simplifying Ratios:**

Just like fractions, ratios can be simplified by dividing both sides of the ratio by their greatest common divisor (GCD).

Example: Simplify the ratio 10:20.

$$\frac{10}{20} = \frac{1}{2} \quad \Rightarrow \quad \text{Simplified Ratio} = 1:2$$

2.15 Proportion:

Proportion is an equation that states that two ratios are equal. It expresses the relationship between two ratios and is typically written as:

$\frac{a}{b} = \frac{c}{d}$This can be read as: "a is to b as c is to d."A proportion tells you that the ratios are equivalent, i.e., the relative size or value of the quantities in one ratio is the same as in the other.

2.16 Types of Proportions:

- **Direct Proportion (or Direct Variation):**

Two quantities are said to be in direct proportion if they increase or decrease together. If one quantity increases, the other does too, and vice versa. Mathematically, if (x)is directly proportional to (y), we can write:

$$\frac{x}{y} = \text{constant}$$

Or equivalently:

$$[x \propto y]$$

Example: If the cost of 5 pens is $10, the cost of 1 pen is in direct proportion to the number of pens. If the price for 5 pens is $10, the price for 1 pen is:

$$\frac{10}{5} = 2 \quad \Rightarrow \quad \text{Cost of 1 pen} = 2$$

- **Inverse Proportion (or Inverse Variation):**

Two quantities are said to be in inverse proportion if one increases while the other decreases. In other words, the product of the two quantities remains constant. If (x)is inversely proportional to (y), we write:

$[x \propto \frac{1}{y}]$ Or equivalently:

$$[x \cdot y = \text{constant}]$$

Example: If 5 workers can complete a task in 10 days, the number of days required to complete the same task by 10 workers would be in inverse proportion to the number of workers:

$$[x \cdot y = \text{constant}]$$

So, with 10 workers, the time taken would be:

$$\frac{50}{10} = 5 \quad \Rightarrow \quad \text{Time taken by 10 workers} = 5 \text{ days}$$

- **Solving Proportions:**

To solve a proportion $(\frac{a}{b} = \frac{c}{d})$,you can use cross-multiplication. This means:

$[a \cdot d = b \cdot c]$Thus, the cross product of the two ratios should be equal.

Example 1: If$(\frac{3}{4} = \frac{x}{8})$, what is (x)?

Solution:

Cross-multiply to solve for x

$$[3 \cdot 8 = 4 \cdot x]$$

$$24 = 4x \quad \Rightarrow \quad x = \frac{24}{4} = 6$$

So, (x = 6).

Example 2: A recipe calls for 3 cups of flour for 4 servings of a cake. How much flour is needed for 10 servings?

Solution:

Let the amount of flour for 10 servings be (x).

The proportion is:

$[\frac{3}{4} = \frac{x}{10}]$Cross-multiply to solve for (x):$[3 \cdot 10 = 4 \cdot x]$

$[30 = 4x \quad \Rightarrow \quad x = \frac{30}{4} = 7.5]$So, you need 7.5 cups of flour for 10 servings.

2.17 Applications of Ratio and Proportion:

Example 1: If the ratio of time taken to travel between two points is in proportion to the ratio of the distances, we can use proportion to find one of the unknowns. For instance, if a car travels 100 miles in 2 hours, how far will it travel in 5 hours at the same speed?

Solution:

Let the distance traveled in 5 hours be x

The proportion will be:

$[\frac{100}{2} = \frac{x}{5}]$Cross-multiply:

$$[100 \cdot 5 = 2 \cdot x]$$

$$[500 = 2x \quad \Rightarrow \quad x = \frac{500}{2} = 250]$$

So, the car will travel 250 miles in 5 hours.

Example 2: A chemist needs to mix a 40% solution with a 60% solution to create 20 litres of a 50% solution. How many litres of each solution should be used?

Solution:

Let x litres of the 40% solution and 20 - x litres of the 60% solution be mixed to create 20 litres of the 50% solution.

The equation based on the concentration of the solutions is:

$$[40x + 60(20 - x) = 50 \times 20]$$

Simplifying:$[40x + 1200 - 60x = 1000]$

$$[-20x + 1200 = 1000]$$

$$[-20x = -200 \quad \Rightarrow \quad x = 10]$$

So, 10 litres of the 40% solution and 10 litres of the 60% solution are needed.

2.18 Partnership:

In business, a partnership refers to an agreement between two or more people to conduct business together and share profits and losses. Each partner contributes to the business either in terms of capital, skill, or labour, and in return, they share the profits or losses in a ratio that is typically pre-agreed upon.

Key Concepts:

1. Capital Contribution: The amount of money or assets that each partner contributes to the business.

2. Profit Sharing Ratio: The ratio in which the partners share the profits or losses of the business. This ratio may or may not be the same as the capital ratio.

3. Loss Sharing Ratio: The ratio in which the partners share any losses of the business. Typically, this is the same as the profit-sharing ratio unless otherwise agreed.

4. Partnership Agreement: A written contract that details the terms of the partnership, including capital contribution, profit-sharing ratio, and other key provisions.

Types of Partnership Contributions:

1. Capital Contribution: The financial investment each partner makes into the business.

2. Skill/Work Contribution: Sometimes a partner contributes their time and skills instead of money (e.g., labour, expertise, etc.).

2.19 Basic Formula for Partnership Problems:

Profit/Loss for Each Partner:

$$\text{Partner's Share} = \frac{\text{Partner's Capital Contribution}}{\text{Total Capital}} \times \text{Total Profit or Loss}$$

Examples of Partnership Problems

Example 1: A and B start a business by investing Rs.30,000 and Rs.20,000 respectively. After one year, the total profit is Rs.10,000. What is the share of the profit for each partner?

Solution:

Capital Contribution:

A = Rs.30,000, B = Rs.20,000

Total Capital = A's Capital + B's Capital = Rs.30,000 + Rs.20,000 = Rs.50,000

Profit Sharing Ratio:

A : B = 30,000 : 20,000 = 3 : 2

Total Profit = Rs.10,000

Now, to find A and B's share of the profit, we use the profit-sharing ratio (3:2).

A's Share of Profit:

$$[\text{A's Profit} = \frac{3}{5} \times 10{,}000 = 6{,}000]$$

B's Share of Profit:

$$[\text{B's Profit} = \frac{2}{5} \times 10{,}000 = 4{,}000]$$

So, A gets Rs.6,000, and B gets Rs.4,000.

Example 2: C, D, and E start a business with investments of Rs.50,000, Rs.30,000, and Rs.20,000, respectively. The total profit at the end of the year is Rs.24,000. The profit-sharing ratio is agreed to be in the ratio of their capital investments. What will be the share of each partner in the profit?

Solution:

Capital Contribution:

C = Rs.50,000, D = Rs.30,000, E = Rs.20,000

Total Capital = C's Capital + D's Capital + E's Capital = Rs.50,000 + Rs.30,000 + Rs.20,000 = Rs.100,000

Profit Sharing Ratio:

C : D : E = 50,000 : 30,000 : 20,000 = 5 : 3 : 2

Total Profit = Rs.24,000

Now, to find C, D, and E's share of the profit, we use the ratio (5:3:2).

C's Share of Profit:

$$[\text{C's Profit} = \frac{5}{10} \times 24{,}000 = 12{,}000]$$

D's Share of Profit:

$$[\text{D's Profit} = \frac{3}{10} \times 24{,}000 = 7{,}200]$$

E's Share of Profit:

$$[\text{E's Profit} = \frac{2}{10} \times 24{,}000 = 4{,}800]$$

So, C gets Rs.12,000, D gets Rs.7,200, and E gets Rs.4,800.

Example 3: F and G start a business by investing Rs.40,000 and Rs.60,000, respectively. F's investment is for 6 months, and G's investment is for 8 months. If the total profit at the end of the year is Rs.18,000, how much profit will each partner get?

Solution:

Capital Contribution:

F = Rs.40,000, G = Rs.60,000

Time Period:

F's investment time = 6 months, G's investment time = 8 months

The formula for profit-sharing when the time periods are involved is to multiply the capital invested by the time the capital was invested.

F's Effective Capital = ($40{,}000 \times 6 = 240{,}000$)

G's Effective Capital = ($60{,}000 \times 8 = 480{,}000$)

Now, we find the total effective capital:

$$[\text{Total Effective Capital} = 240{,}000 + 480{,}000 = 720{,}000]$$

The ratio of their capital contributions:

$$[\text{F's Share:G's Share} = 240{,}000 : 480{,}000 = 1 : 2]$$

Total Profit = Rs.18,000

Now, using the ratio (1:2), we divide the profit:

F's Share of Profit:

$$[\text{F's Profit} = \frac{1}{3} \times 18{,}000 = 6{,}000]$$

G's Share of Profit:

$$[\text{G's Profit} = \frac{2}{3} \times 18{,}000 = 12{,}000]$$

So, F gets Rs.6,000 and G gets Rs.12,000.

Example 4: H and I start a partnership business. H invests Rs.10,000 for the entire year, while I invests Rs.15,000 for 6 months and then withdraws half of his capital. The total profit at the end of the year is Rs.30,000. What is the share of the profit for each partner?

Solution:

Capital Contribution:

H = Rs.10,000 (invested for the full year),

I = Rs.15,000 for 6 months, then Rs.7,500 for the remaining 6 months.

To find the total effective capital, we calculate the contribution of each partner in terms of capital multiplied by the time the capital was invested.

H's Effective Capital = $(10{,}000 \times 12 = 120{,}000)$

I's Effective Capital = $(15{,}000 \times 6 = 90{,}000)$(for the first 6 months)

Then, I withdrew half of his capital, so the next 6 months, his capital was Rs.7,500:

$$[\text{I's Effective Capital for 6 months} = 7{,}500 \times 6 = 45{,}000]$$

Total effective capital of I:

$$[\text{I's Total Effective Capital} = 90{,}000 + 45{,}000 = 135{,}000]$$

Total effective capital of both partners:

$$[\text{Total Effective Capital} = 120{,}000 + 135{,}000 = 255{,}000]$$

Now, the ratio of their contributions is:

$$[\text{H's Share:I's Share} = 120{,}000 : 135{,}000 = 8 : 9]$$

Total Profit = Rs.30,000

Now, using the ratio (8:9), we divide the profit:

H's Share of Profit:

$$[\text{H's Profit} = \frac{8}{17} \times 30{,}000 = 14{,}117.64]$$

I's Share of Profit:

$$[\text{I's Profit} = \frac{9}{17} \times 30{,}000 = 15{,}882.36]$$

So, H gets Rs.14,117.64, and I gets Rs.15, 882.36.

2.20 Time and Work:

Time and work are a mathematical concept that deals with calculating the time required to complete a task based on the rate at which work is done, either by individuals or groups. It helps determine how long a task will take when performed by people working alone or together, considering their efficiency and productivity rates. Widely used in project management, engineering, and other fields, time and work calculations allow for effective resource allocation, scheduling, and performance optimization.

2.21 Basic Concepts:

In solving the problems based on time and work, we need to calculate the following parameters.

(A) **Time: -** Time taken to complete an assigned job.

(B) **Individual time: -** Time needed by single person to complete a job.

(C) **Work: -** It is the amount of work done.

Work done = Efficiency × Time taken

Example 1: A contractor is hired to complete a project in 40 days. After working alone for 10 days, the contractor hires an assistant, whose efficiency is half that of the contractor. Together, they complete the project in the remaining 20 days. Calculate the number of days the contractor would have taken to complete the project alone.

Solutions: Contractor's Efficiency: If the contractor can complete the project in x days, then their efficiency is $\frac{1}{x}$ (i.e., the fraction of the project completed per day).

Assistant's Efficiency: The assistant's efficiency is half of the contractor's, which is $\frac{1}{2x}$.

Work Done Alone: The contractor works alone for 10 days:

Work done by contractor in 10 days $= \frac{10}{x}$

Remaining Work: The remaining work after 10 days is:

Remaining work$= 1 - \frac{10}{x} = \frac{x-10}{x}$

Combined Efficiency: When both the contractor and the assistant work together, their combined efficiency is:

Combined efficiency $= \frac{1}{x} + \frac{1}{2x} = \frac{3}{2x}$

Work Done Together: In the next 20 days, they complete:

$$\text{Work done together} = \frac{3}{2x} \times 20 = \frac{30}{x}$$

Setting up the Equation: Since they complete the remaining work in those 20 days:

$$\frac{30}{x} = \frac{x-10}{x}$$

Simplifying the Equation: Multiplying through by x :

30 = x - 10

x = 40

Thus, the contractor would have taken 40 days to complete the project alone.

Example 2: Two machines, A and B, can complete a production task in 12 and 16 hours, respectively. Due to maintenance, Machine A operates at only 75% efficiency. Calculate the time taken for both machines to complete the task working together with A at reduced efficiency.

Solutions: Machine A can complete the task in 12 hours, so its full efficiency is:

$\text{Efficiency of A (full)} = \frac{1}{12}\text{tasks per hour.}$

Machine B can complete the task in 16 hours, so its efficiency is:

$$\text{Efficiency of B} = \frac{1}{16}\text{tasks per hour}$$

Due to maintenance, Machine A operates at 75% efficiency:

$\text{Efficiency of A (reduced)} = 0.75 \times \frac{1}{12} = \frac{0.75}{12} = \frac{1}{16}\text{tasks per hour}$

$\text{Combined Efficiency} = \text{Efficiency of A (reduced)} + \text{Efficiency of B} = \frac{1}{16} + \frac{1}{16} = \frac{2}{16}$
$= \frac{1}{8}\text{tasks per hour}$

$\text{Time Taken} = \frac{1}{\text{Combined Efficiency}} = \frac{1}{\frac{1}{8}} = 8\text{hours} = 480\text{minutes}$

The time taken to complete one task is the reciprocal of the combined efficiency:

$\text{Time Taken} = \frac{1}{\text{Combined Efficiency}} = \frac{1}{\frac{1}{8}} = 8\text{hours} = 480\text{minutes}$

Example 3: Three workers, X, Y, and Z, can complete a task individually in 15, 20, and 30 hours, respectively. If they all start working together, but Z leaves after 2 hours, how long will it take for X and Y to complete the remaining work?

Solutions: Worker X can complete the task in 15 hours.

Worker Y can complete the task in 20 hours.

Worker Z can complete the task in 30 hours.

$\text{Efficiency of X} = \frac{1}{15}$, $\text{Efficiency of Y} = \frac{1}{20}$, $\text{Efficiency of Z} = \frac{1}{30}$ $\text{Combined Efficiency} = \text{Efficiency of X} + \text{Efficiency of Y} + \text{Efficiency of Z} = \frac{1}{15} + \frac{1}{20} + \frac{1}{30} == \frac{4}{60} + \frac{3}{60} + \frac{2}{60} = \frac{9}{60} = \frac{3}{20}\text{(tasks per hour)}$

In 2 hours, the total work done by X, Y, and Z together is:

$\text{Work Done} = \text{Combined Efficiency} \times 2 = \frac{3}{20} \times 2 = \frac{3}{10}$ of the task.

The total work is considered as 1 task, so the remaining work after 2 hours is:

$\text{Remaining Work} = 1 - \frac{3}{10} = \frac{7}{10}$ of the task

Now, X and Y will continue working together. Their combined efficiency is:Efficiency of X and Y $= \frac{1}{15} + \frac{1}{20} = \frac{4}{60} + \frac{3}{60} = \frac{7}{60}$ (tasks per hour)

Let t be the time taken in hours to complete the remaining work:

$t \times \text{Combined Efficiency} = \text{Remaining Work} = t \times \frac{7}{60} = \frac{7}{10} t = \frac{7/10}{7/60} = \frac{7}{10} \times \frac{60}{7} = \frac{60}{10} = 6\text{hours}$

2.22 Applications of Time and Work:

1. **Project Management:** Used to schedule tasks, assign resources, and calculate timelines for project completion.
2. **Engineering and Manufacturing:** Essential for estimating production times, managing maintenance schedules, and coordinating multi-team efforts.
3. **Labor Productivity Analysis:** Helps assess individual and team efficiency, identify performance bottlenecks, and plan workforce allocation.
4. **Cost Estimation:** Used to estimate labour costs by predicting the time needed for job completion.
5. **Supply Chain and Logistics:** Supports planning for logistics, optimizing shipping times, and ensuring timely delivery across the supply chain.

2.23 Time and Distance:

The concepts of time and distance are foundational in physics and mathematics, often explored in various contexts, from basic kinematics to advanced applications in astrophysics and cosmology.

2.24 Basic Concepts:

1. **Speed, Velocity, and Acceleration:**

- **Speed:** is the rate of change of distance with respect to time (distance/time).
- **Velocity:** is a vector quantity that includes both speed and direction.
- **Acceleration:** is the rate of change of velocity with respect to time.

2. Kinematic Equations (Newton's equation of motion):

$$(i) v = u + at$$

$$(ii) s = ut + \frac{1}{2}at^2$$

$$(iii) v^2 = u^2 + 2as$$

where

- v = final velocity
- u = initial velocity
- a = acceleration
- t = time
- s = distance

Example 1: Two trains are moving towards each other on parallel tracks. Train A is traveling at 90 km/h, and Train B is traveling at 60 km/h. They are initially 300 km apart.

Solutions: Train A is traveling at 90 km/h, and Train B is traveling at 60 km/h. They are initially 300 km apart.

To find the time it takes for them to meet, we first calculate their combined speed

90 km/h + 60 km/h = 150 km/h.

Next, we use the formula for time, which is distance divided by speed:

Time = Distance / Speed = 300 km / 150 km/h = 2 hours.

Therefore, the two trains will meet after 2 hours.

Example 2: A ball is thrown upward with an initial speed of 15 m/s. Calculate the maximum height it reaches, and the time taken to reach that height.

Solutions: The acceleration due to gravity is $g = -9.81$ m/sec^2.

At maximum height, the final velocity $v = 0 m/s$.

Using the equation $v = u + at$:

$$0 = 15 + (-9.81)t \Longrightarrow 9.81t = 15 \Longrightarrow t = \frac{15}{9.81} \approx 1.53s.$$

Now, using the equation for maximum height $s = ut + \frac{1}{2}at^2$:

$$s = 15(1.53) + \frac{1}{2}(-9.81)(1.53)^2.$$

Calculating this gives:

$s \approx 22.95 - \frac{1}{2} \times 9.81 \times 2.3409 \approx 22.95 - 11.49 \approx 11.46\text{m}.$

Thus, the maximum height reached by the ball is approximately 11.46 meters, and the time taken to reach that height is approximately 1.53 seconds.

Example 3: A train starts from rest and accelerates uniformly at 3 m/sec² for 20 seconds. How far does it travel during this time?

Solutions: To find the distance traveled by a train that starts from rest and accelerates uniformly, we can use the equation:

$$s = ut + \frac{1}{2}at^2$$

s is the distance traveled, u is the initial velocity (0 m/s, since the train starts from rest), a is the acceleration (3 m/s²), t is the time (20 seconds).

Substituting the values into the equation:

$$s = 0 \times 20 + \frac{1}{2} \times 3 \times (20)^2 s = 0 + \frac{1}{2} \times 3 \times 400 s = \frac{3}{2} \times 400 s = 600\text{m}$$

Therefore, the train travels 600 meters during this time.

2.25 Application of Time and Distance:

1. **Astrophysics and Cosmology:** Time and distance are fundamental in measuring vast cosmic scales and understanding the dynamics of celestial bodies. Astrophysicists use various methods to determine distances to stars and galaxies, such as parallax, redshift, and standard candles (like supernovae).
2. **Navigation and GPS Technology:** GPS technology relies on satellites that communicate their positions and the time it takes for signals to reach a receiver. By calculating the time delay, the GPS receiver can determine its distance from several satellites and triangulate its exact position on Earth.
3. **Engineering: Structural Analysis:** Engineers analyse time and distance in the context of loads, forces, and structural integrity. They calculate how long structures like bridges can withstand forces from vehicles, people, and environmental factors.
4. **Sports Science: Motion Analysis:** In sports science, analysing time and distance helps in optimizing athlete performance. Coaches use motion analysis to assess speed, acceleration, and overall biomechanics.

5. **Telecommunications: Signal Propagation:** Telecommunications systems rely on the principles of time and distance to ensure efficient data transmission. Understanding how signals propagate over distances helps in designing networks and optimizing communication protocols.

2.26 Problems on Trains:

Problems related to trains typically engage with the fundamental principles of kinematics, particularly the interrelationship between speed, distance, and time in the context of linear motion. These problems frequently focus on the concept of relative speed, which is contingent upon the direction of motion of the trains—whether they are approaching each other or moving apart.

2.26 Key problem types include:

1. **Meeting Point Calculations:** This involves determining the time required for two trains, initiating from distinct points and traveling towards one another at constant speeds, to converge at a specific location. The solution employs the additive property of relative speed.
2. **Passing Time Analysis:** This entails calculating the duration for which one train completely overtakes another train or a stationary object, necessitating the consideration of both trains 'lengths and their relative velocity.
3. **Crossing Obstacles**: This problem focuses on the temporal duration a train requires to traverse platforms, bridges, or other stationary structures, factoring in both the length of the train and the dimensions of the obstacle.

Such problems serve as practical applications of classical mechanics and provide insights into the mathematical relationships governing motion, thereby enhancing the understanding of dynamic systems in a railway context.

Example 1: Train A leaves Station X and travels towards Station Y at a speed of 75 km/h. Simultaneously, Train B departs from Station Y towards Station X at a speed of 90 km/h. If the distance between Station X and Station Y is 300 km, how long will it take for the two trains to meet?

Solutions: $\text{Relative Speed} = \text{Speed of Train A} + \text{Speed of Train B} = 75\text{km/h} + 90\text{km/h} = 165\text{km/h}$

$$\text{Time} = \frac{\text{Distance}}{\text{Relative Speed}} = \frac{300\text{km}}{165\text{km/h}} \approx 1.82\text{hours} \approx 1\text{hour and} 49\text{minutes}$$

Example 2: Train A is 250 meters long and moves at a speed of 60 km/h. Train B, which is 150 meters long, moves at a speed of 90 km/h in the opposite direction. How long will it take for Train A to completely pass Train B?

Solutions: Speed of Train A: $60\text{km/h} = \frac{60\times1000}{3600} \approx 16.67\text{m/s}$

Speed of Train B: $90\text{km/h} = \frac{90\times1000}{3600} \approx 25\text{m/s}$

Calculate the relative speed: $\text{Relative Speed} = 16.67\text{m/s} + 25\text{m/s} = 41.67\text{m/s}$

Find the total length of the trains: $\text{Total Length} = 250\text{m} + 150\text{m} = 400\text{m}$

$$\text{Time} = \frac{\text{Total Length}}{\text{Relative Speed}} = \frac{400\text{m}}{41.67\text{m/s}} \approx 9.6\text{seconds}$$

Example 3: A train 400 meters long is moving at a speed of 90 km/h. How long will it take to cross a platform that is 600 meters long.

Solutions: Convert speed to meters per second: $90\text{km/h} = \frac{90\times1000}{3600} \approx 25\text{m/s}$

Calculate the total distance to be covered (length of the train plus length of the platform):$\text{Total Distance} = 400\text{m} + 600\text{m} = 1000\text{m}\text{Time} = \frac{\text{Total Distance}}{\text{Speed}} = \frac{1000\text{m}}{25\text{m/s}} = 40\text{seconds}$

2.27 Simple Interest and Compound Interest:

- **Simple Interest:** Simple interest can be defined as the principal amount of a loan or deposit a person makes into their bank account.

- **Compound Interest:** Compound interest is the interest that accumulates and compounds over the principal amount.

2.28 Difference between Simple and Compound Interest:

The primary distinction between simple interest and compound interest lies in their calculation methodologies. Simple interest is calculated exclusively on the principal amount, whereas compound interest considers both the principal and the accumulated interest over specified compounding periods. These two concepts are fundamental in various financial services, particularly in the banking sector. Simple interest is typically applied to loans such as instalment loans, auto loans, educational loans, and mortgages. Conversely, compound interest is predominantly utilized in savings accounts and investment vehicles, as it effectively increases the interest earned by accounting for interest on both the principal and any previously accrued interest.

Example 1: A student takes out a loan of Rs. 5,000 at a simple interest rate of 6% per annum for 4 years. How much interest will the student pay over the life of the loan, and what will be the total amount to be repaid?

Solutions: To calculate the interest paid on the loan, use the formula for simple interest:

Interest (I) = Principal (P) × Rate (R) × Time (T)

Where:
- Principal (P) = Rs. 5,000
- Rate (R) = 6% per annum = 0.06
- Time (T) = 4 years

Substituting the values into the formula:

I = 5000 × 0.06 × 4

I = 5000 × 0.24

I = Rs.1,200

The total amount to be repaid is the sum of the principal and the interest:

Total Amount = Principal + Interest

Total Amount = 5000 + 1200

Total Amount = Rs. 6,200

The student will pay Rs. 1,200 in interest over the life of the loan, and the total amount to be repaid is ₹6,200.

Example 2: An investment ofRs2,0000 is made in a savings account that pays an annual interest rate of 5%, compounded quarterly. How much will the investment be worth after 5 years?

Solutions: To calculate the future value of an investment compounded quarterly, use the formula for compound interest:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where:

- A = the future value of the investment/loan, including interest
- P = the principal investment amount (₹200,000)
- r = the annual interest rate (decimal) (5% = 0.05)
- n = the number of times that interest is compounded per year (quarterly = 4)
- t = the number of years the money is invested (5 years)

Substituting the values into the formula:

$$A = 200000\left(1 + \frac{0.05}{4}\right)^{4\times5}$$

Calculating the values step by step:

Calculate the quarterly interest rate:

$$\frac{0.05}{4} = 0.0125$$

Calculate the total number of compounding periods:

$$4 \times 5 = 20$$

Substitute these values into the formula:

$$A = 200000(1 + 0.0125)^{20}$$

$$A = 200000(1.0125)^{20}$$

$$A \approx 256407.45$$

The investment will be worth approximately Rs. 256,407.45 after 5 years.

Example 3: You have the option to invest Rs.15,000 in either a simple interest account at 4% per annum for 5 years or a compound interest account at 3.5% per annum compounded annually for the same period. Which option yields a higher return?

Solutions: For the simple interest account:

Use the formula for simple interest:

$I = P \times R \times T$

Where:
- P = Rs.15,000 (Principal)
- R = 4% = 0.04 (Rate)
- T = 5 years (Time)

Calculating the interest:

$I = 15000 \times 0.04 \times 5$

$I = 15000 \times 0.20$

I = Rs.3,000

Now, calculate the total amount:

Total Amount from Simple Interest = Principal + Interest

= 15000 + 3000 = Rs.18,000

For the compound interest account:

Use the formula for compound interest:

$A = P(1 + r)^t$

Where:
- P = Rs.15,000 (Principal)
- r = 3.5\% = 0.035 (Rate)
- t = 5 years (Time)

Calculating the total amount:

$A = 15000(1 + 0.035)^5 = 15000(1.035)^5$

$A \approx 15000 \times 1.188677$= Rs.17,830.15

Now we can compare the two options:

- Total Amount from Simple Interest: Rs.18,000
- Total Amount from Compound Interest: Rs.17,830.15

The simple interest option yields a higher return of Rs.18,000 compared to Rs.17,830.15 from the compound interest option.

2.29 Application of Simple and Compound Interest:

1. **Personal Loans**: Simple interest is commonly used in personal loans, such as auto loans or short-term personal loans. Borrowers can easily calculate the total amount of interest payable, allowing for straightforward financial planning and budgeting.

2. **Education Loans:** Many educational loans apply simple interest during the study period. This method simplifies the calculation of interest owed while students are still in school, easing their financial burden until they begin repayment.

3. **Short-Term Investment Instruments**: Certain short-term investments, such as fixed deposits with shorter durations, may use simple interest. This approach allows investors to predict their returns with clarity over the investment period.

4. **Late Payment Penalties**: Businesses often impose simple interest as a penalty for late payments. This ensures a transparent calculation of additional charges, helping to motivate timely payments from clients and customers.

5. **Insurance Premiums**: Some insurance policies may calculate the interest on premiums using simple interest, particularly in cases of one-time premium payments, making it easier for policyholders to understand their financial commitments.

6. **Savings Accounts**: Compound interest is widely used in savings accounts, allowing depositors to earn interest on both their principal and accumulated interest. This compounding effect significantly enhances the growth of savings over time.

7. **Investment Growth**: In investments such as stocks, mutual funds, and real estate, compound interest plays a vital role. Reinvesting dividends and capital gains can substantially increase the overall returns on investment, benefiting long-term investors.

8. **Retirement Planning**: Retirement accounts, including 401(k)s and IRAs, utilize compound interest to maximize growth. By allowing contributions to grow through compounding, individuals can accumulate significant savings for their retirement years.

9. **Credit Cards**: Credit card companies often charge compound interest on outstanding balances. Consumers who comprehend the implications of compounding can make more informed decisions about managing their credit card debt and avoiding high-interest charges.

10. **Financial Modelling and Forecasting**: Financial analysts use compound interest calculations to project future cash flows, assess investment viability, and develop strategic financial plans. This analytical approach aids businesses and individuals in making data-driven decision.

UNIT-III

Area, Volume and Surface area, Polygons, True Discount, Banker’s Discount, Calendar, Clock, Pie Chart, Line Chart and Bar Diagrams.

3.1 Surface area:

Surface area can be easily calculated through already established formulas. The objects that are mainly focused on are cube, cuboid, sphere, cone and cylinder. The object can also be a combination of two objects.

Example: cone on top of a cylinder.

3.2 Total surface area Vs lateral or curved surface area:

The overall surface area is the region that includes the base(s) and the curving part. It refers to the space covered by an object's surface. If the shape has a curved base and surface, the overall area will equal the sum of the two regions. "The overall area covered by an object, including its base as well as the curving part," says the definition. "If an object contains both a base and a curved region, the total surface area will be equal to the sum of the two."

Except for its centre, the area of a curved surface equals the area of the curved component of the shape (s). For shapes like cones, it is the lateral surface area. "The area that comprises only the curved area of an object or the lateral area of an object by removing the base area of an object," states the explanation. The curved surface area is another name for lateral surface area.

3.3 Surface area of a cube:

It is the total area covered by all six sides of a cube. It is known that area of a square is a^2 and the sides of a cube are squares. Lateral surface area includes four sides and total surface area includes six sides. The surface area of a cube is determined by the length of the side of the given cube.

Therefore,

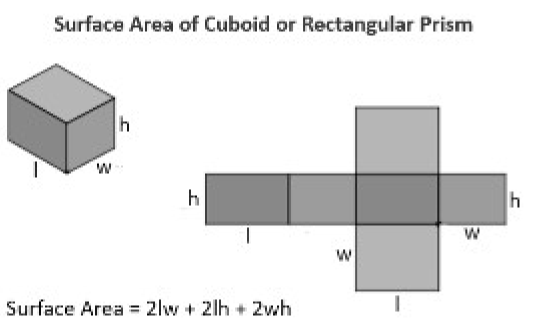
CSA of a cube = $4a^2$

TSA of a cube = $6a^2$

3.4 Surface area of a cuboid:

Unlike cubes, cuboids have length, height and breadth as their dimensions. Change in any dimension results in change in the value of the surface area.

A cuboid also has 6 faces and the total surface area is derived by adding the area of all sides. The 6 faces are divided into 3 rectangles repeated twice.



Area of a rectangle formula is the multiplication of its two sides.

So therefore,

Area of rectangle 1 and 3 = h x l

Area of rectangle 2 and 4 = h x w

Area of rectangle 5 and 6 = l x w

Therefore total surface area of a cuboid = 2(hl + hw+lw)

Lateral surface area is the surface area of 4 sides, i.e. 6 sides of a cuboid – the top and bottom rectangles.

The top and bottom rectangles area = 2(l x w)

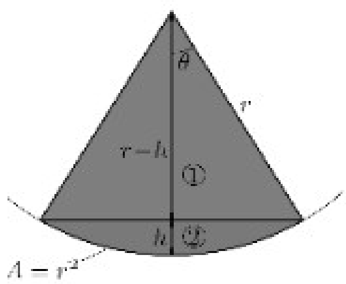
Therefore, lateral surface area = 2(hl + hw+lw) – 2(lw)

= 2(hl+hw)

= 2h(l+w)

3.5 Area of a cone:

The cone is a complex object. However, finding its area is relatively easy. To find the surface area of a cone, one must be familiar with Pythagoras theory. The cone is shaped with a circle at its base and a triangular top. The height is determined by the centre point of the circle to the tip of the cone which is denoted by the letter h. Length is the length of one side of the triangular part denoted by letter l and radius is the radius of the circle at the base, denoted by letter r.



By applying pythagoras theorem, we know that $h^2 + r^2 = l^2$

$l = \sqrt{h^2+r^2}$

Therefore total surface area $\pi = r\,(r + l)$

$= \pi\, r(r+\sqrt{h^2+ r^2})$

Curved surface area = πrl

$= \pi r(h^2+r^2)$

3.6 Area of a cylinder:

The surface area of a cylinder can be determined by adding the curved surface of a cylinder with the flat surface of a cylinder.

There are two bases in a cylinder and it is circle in shape.

The curved surface area of a cylinder = $2\pi rh$.

Where h is the height and r is the radius.

Total surface area of a cylinder = Area of two bases + curved surface are

$$= 2\pi r^2 + 2\pi rh$$

3.7 Area of a sphere:

A sphere is a three dimensional object. The 2D form of a sphere is a circle.

Example, a globe or a ball is a sphere.

A sphere is related to curved surfaces like the cylinder. The radius of a cylinder and a sphere is the same and the height is also the same. The height to a sphere is called the diameter. A sphere can perfectly fit into the cylinder,

Therefore, lateral surface area of a cylinder = total surface area of a sphere.

Lateral surface area of a cylinder = $2\pi rh$.

$h = 2r$ (since height = diameter, diameter = 2x radius.)

Surface area of sphere = $2\pi r(2r) = 4\pi r^2$.

The curved surface area of the sphere = total surface of the sphere as there are no flat surfaces in a sphere.

3.8 Volume:

It is defined by the capacity to hold by a three dimensional object. Volume can highly differ from object to object. It is to be remembered that empty space in an object does not solely amount to holding capacity. A solid object like an eraser also has volume in it.

Volume is identified using various formulas for different objects like, cube, cuboid, sphere, cylinder and cone.

3.9 Volume of a cube:

A cube has equal sides and volume is usually determined by multiplying the height, width and length. All sides of a cube are denoted by a.

Therefore volume = a *a*a

= a^3 units.

3.10 Volume of a cuboid:

A cuboid has length, breadth and height. A cuboid can also be said as a stack of rectangles.

Therefore its volume is = l x b x h units

3.11 Volume of a cylinder:

Like a cuboid, a cylinder is the stack of circles. There are 2 bases to a cylinder and is distanced by height called h.

Volume of a cylinder = $\pi r^2 h$ units

That is the area of a circle multiplied by the height.

3.12 Volume of a cone:

It is derived by one-third of the cone with the area of circular base and the height.

Volume of a cone = $1/3\ r^2\ h$ units

Volume of a cone with height and diameter = $(1/12)\pi d^2 h$ (since $r=d/2$) units

Volume of a cone with slant height = $(1/3)\pi r^2 h = (1/3)\pi r^2\sqrt{(L^2 - r^2)}$. units

$h = \sqrt{(L^2 - r^2)}$

3.15 Volume of a sphere:

Volume of a sphere with radius r = $4/3\ \pi r^3$ units.

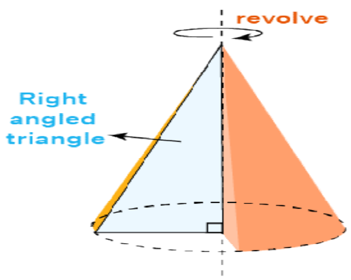
3.16 Right Circular Cone:

A right circular cone is a type of cone whose axis falls perpendicular on the plane of the base. A cone is a 3D geometric figure that has a flat circular surface and a curved surface that meet at a point toward the top. The point formed at the end of the cone is called the apex or vertex, whereas the flat surface is called the base. Any triangle will form a cone when it is rotated, taking one of its two short sides as the axis of rotation.

- **What is a Right Circular Cone?**

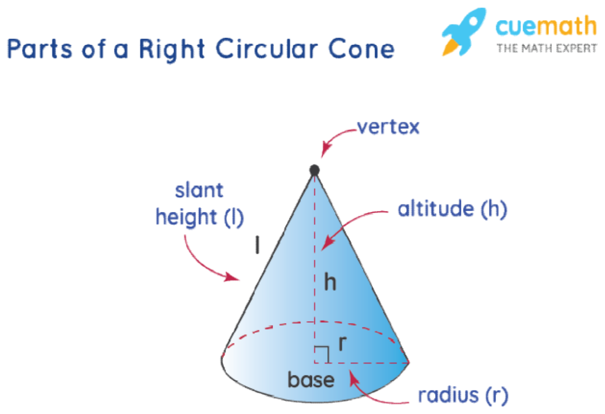
A right circular cone is a type of cone with an axis perpendicular to the plane of the base. A right circular cone is generated by a revolving right triangle about one of its legs. We can also observe this from the figure given below, the right-angled triangle when revolved results in the formation of a cone. The base of a right circular cone is in the shape of a circle.

- **Right circular Cone:**



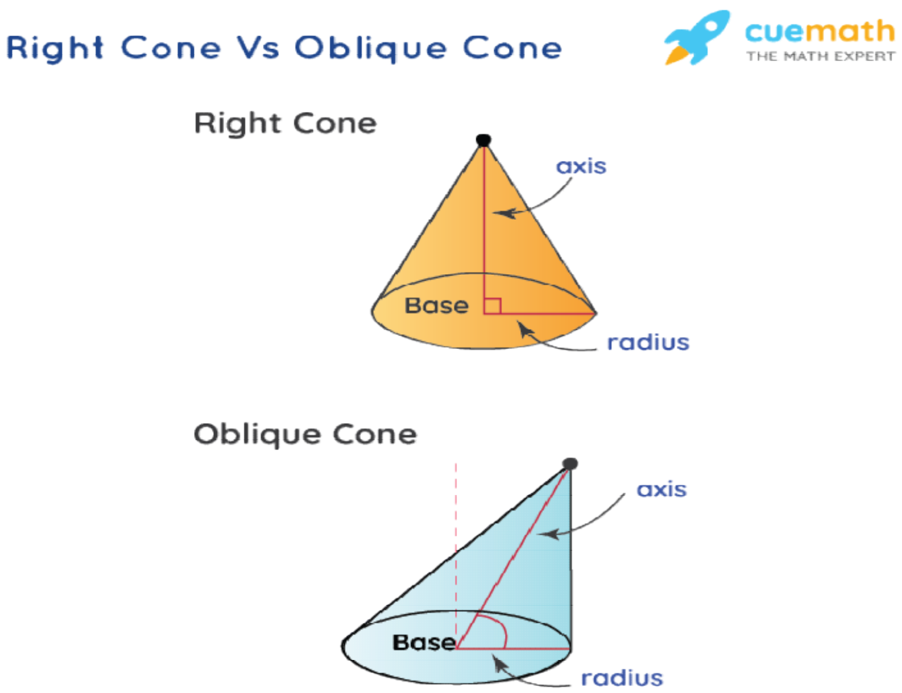
- **Parts of a Right Circular Cone:**

The three elements that are present in the right circular cone are its radius, height, and slant height. The distance between the center of the circular base to any point on the circumference of the base of the right circular cone is defined as its radius, while the distance from the apex of the cone to the center of the circular base is called its height. The distance between the apex of the cone to any point on the circumference of the cone refers to as its slant height.

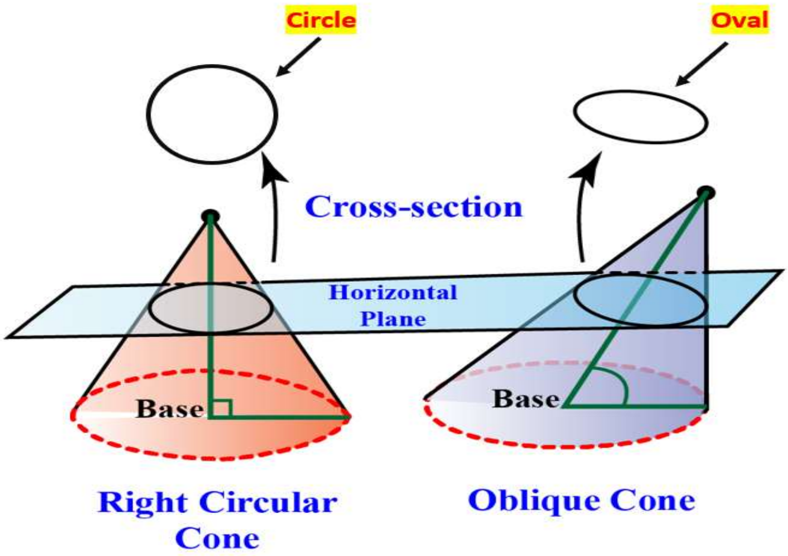


- **Right Circular Cone vs Oblique Cone:**

A cone can be classified into two types based on the alignment of the apex in comparison to the base: the right circular cone and the oblique cone. A right circular cone or regular cone's axis is perpendicular to its base, whereas the oblique cone appears to be tilted and its axis is not perpendicular to the base.



Another way to check if a cone is a right circular cone is to check its cross-section in a horizontal plane. A right circular cone will give a circular cross-section, whereas an oblique cone will give an oval cross-section.



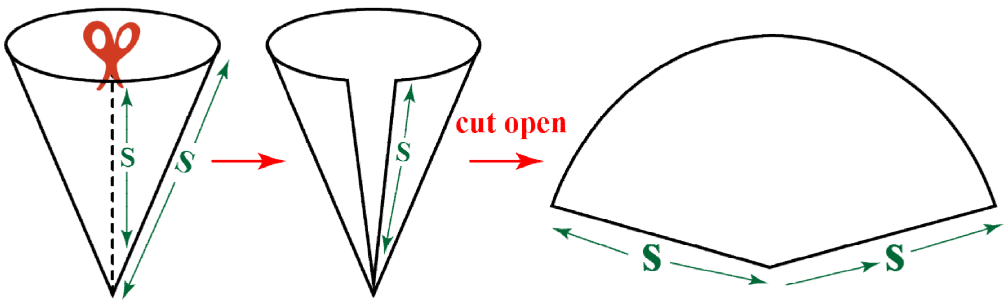
3.17 Properties of a Right Circular Cone:

There are certain properties of a right circular cone that distinguish it from other shapes. These properties are as listed below,

- It has a circular base. The axis is a line that joins the vertex to the center of the base.
- The slant height of the cone is measured from the vertex to the edge of the circular base. It is denoted by 'l' or 's'.
- The altitude or height of a right cone coincides with the axis of the cone and is represented by 'h'.
- If a right triangle is rotated with the perpendicular side as the axis of rotation, a right circular cone is constructed. The surface area generated by the hypotenuse of the triangle is the curved surface or the lateral surface area.
- Any horizontal section of the right circular cone parallel to the base produces the cross-section of a circle.

3.18 Surface Areas of a Right Circular Cone:

The surface area of a right circular cone is defined as the total region covered by the surface of the 3-D dimensional shape. It is expressed using square units, like cm^2, m^2, in^2, ft^2, etc. Let us cut and open up a right circular cone, to understand about the surface area. The curved surface forms a sector with radius 's', as shown below.



The surface area of a right circular cone can be of two types:

- Curved surface area or CSA
- Total surface area or TSA

Curved Surface Area of a Right Circular Cone

The curved surface area of a right circular cone is defined as the region occupied by the curved surface of the cone. We thus don't include the area of the base while referring to the curved surface area of a right circular cone. Curved surface area is also known as the lateral surface area.

Total Surface Area of a Right Circular Cone

The total surface of a right circular cone is defined as the region or area of the complete surface of the cone, including the base area. Let us understand the formulas to calculate the both CSA and TSA of a right circular cone in the next section.

3.19 Surface Area of a Right Circular Cone Formula:

We discussed in the previous section that a right circular cone can have two surface areas, curved surface area or total surface area. We can calculate TSA and CSA for a right circular cone using different formulas.

Curved Surface Area Formula

The formula to calculate a right circular cone's CSA formula can be given as,
The curved surface area of a cone = Area of the sector with radius length equal to the slant height i.e., 's',
Curved surface area of a cone = $\pi rs = \pi r\sqrt{(r^2 + h^2)}$
where,

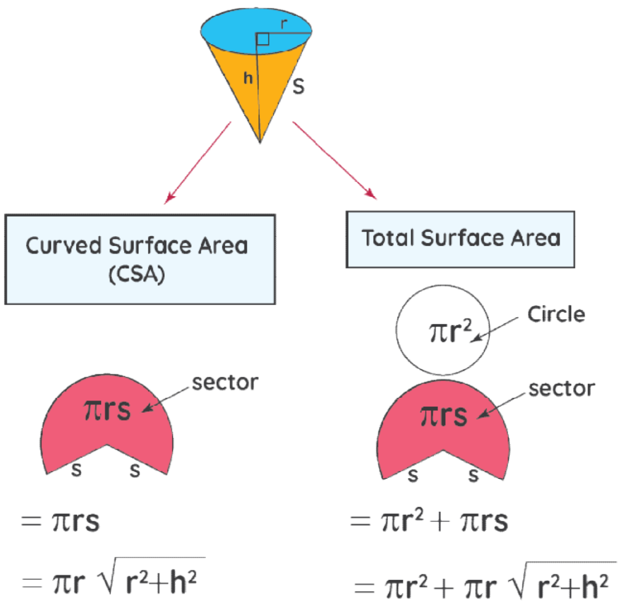
- r = Base radius
- h = Height of right circular cone
- s = Slant height of the right circular cone

Total Surface Area Formula

The formula to calculate a right circular cone's TSA formula can be given as,
The total surface area of a cone = Area of circular base + Curved surface area of a cone (sector's area)
Total surface area of a cone = $\pi r^2 + \pi rs$
Total surface area of a cone = $\pi r^2 + \pi r\sqrt{(r^2 + h^2)}$
where,

- r = Base radius
- h = Height of right circular cone
- s = Slant height of right circular cone

3.20 Surface Area of right circular cone:



Note: Total surface area of a right circular sometimes can be referred to as only surface area. So, whenever we are asked to calculate the surface area, it means we have to find the total surface area of the given cone.

3.21 Volume of a Right Circular Cone:

Volume of the right circular cone is defined as the total space occupied by the object in a 3-dimensional plane. The volume of a cone is expressed in cubic units, like in^3, m^3, cm^3, etc. The volume of a right circular cone that has a circular base with radius 'r' and height 'h' will be equal to one-third of the product of the area of the base and its height. We can calculate the volume of the right circular cylinder, given the base radius and height, using the general formula.

3.22 Volume of a Right Circular Cone Formula:

The volume of a right circular cone can be calculated using the base radius and height of the cone. We can observe from the image given below that the volume of a right circular cone is (1/3) times the volume of a right circular cylinder. The formula to find the volume of the right circular cone can be given as,

Volume of a Cone(V) = (1/3) × Area of Circular Base × Height of the Cone

$V = (1/3) \times \pi r^2 \times h$

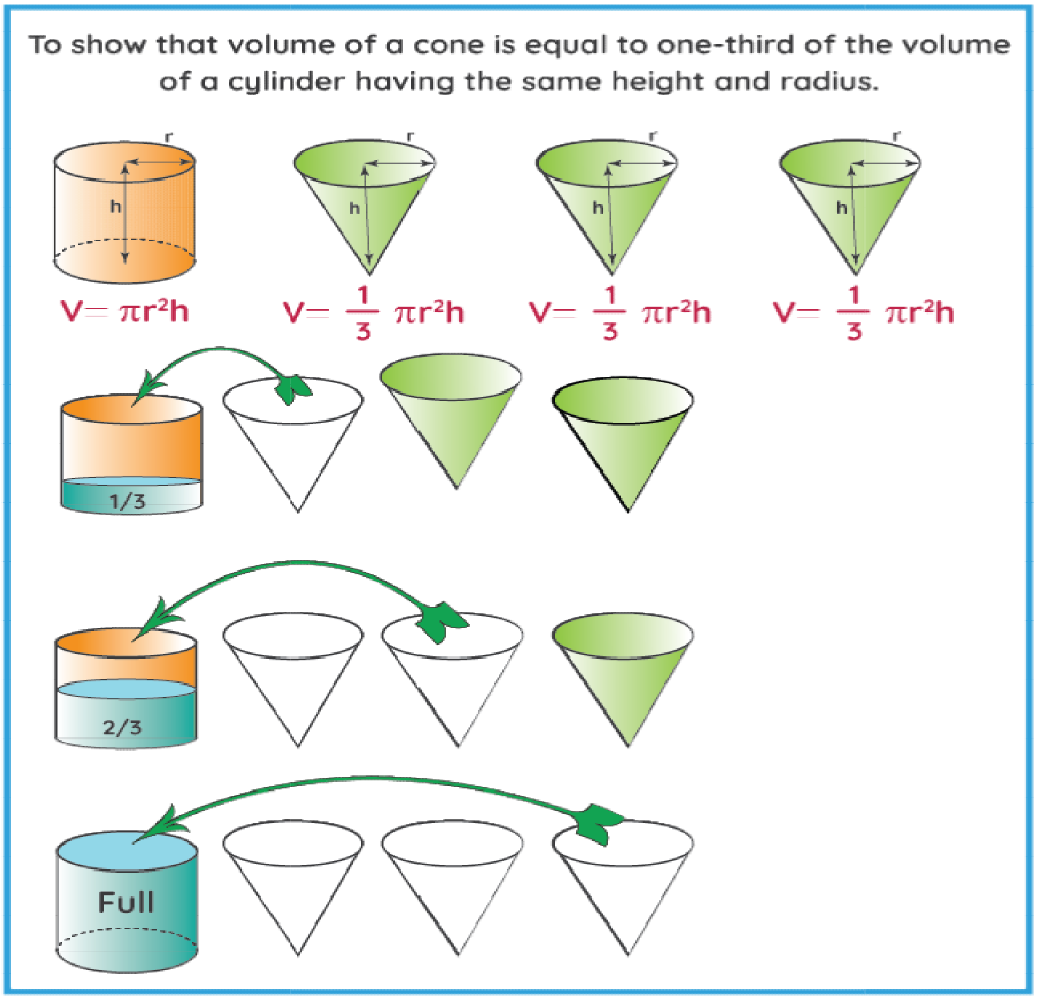
or,

$V = (1/12) \times \pi d^2 \times h$

where,

- r = Base radius
- d = Diameter of base
- h = Height of right circular cone

- **Volume of Right circular cone:**



Solved Examples:

Example 1:

What is the surface area of a cuboid with length, width and height equal to 4.4 cm, 2.3 cm and 5 cm, respectively?

Solution:

Given, the dimensions of cuboid are:

length, l = 4.4 cm

width, w = 2.3 cm

height, h = 5 cm

Surface area of cuboid = 2(wl+hl+hw)

= 2·(2.3 x 4.4 + 5 x 4.4 + 5 x 2.3)

= 87.24 square cm.

Example 2:

What is the volume of a cylinder whose base radii are 2.1 cm and height is 30 cm?

Solution:

Given,

Radius of bases, r = 2.1 cm

Height of cylinder = 30 cm

Volume of cylinder = $\pi r^2 h = \pi \cdot (2.1)^2 \cdot 30 \approx 416$.

Example 3: Suraj wants to buy wrapping paper to cover a cardboard box to pack a present. If the cardboard box has the length, breadth, and height of 20 cm, 30 cm, and 15 cm, how much square wrapping paper of 20 cm should Suraj get?

Solution: We are given the dimensions of the cardboard box and the wrapping paper,

From the question,

length, l = 20 cm,

breadth, b = 30 cm,

height, h = 15 cm,

The total surface area of the cardboard box,

A = 2(l.b + b.h + h.l)

A = 2(20.30 + 30.15 + 15.20)

A = 2(600 + 450 + 300)

A = 2(1350)

A = 2700 cm2

The area of the wrapping paper,

A = 20 x 20

A = 400 cm2

The number of wrapping paper required is given by,

= (surface area of the box)(area of the wrapping paper)

= 2700400

= 6.75 ≈ 7 wrapping papers.

Example 4: Find the curved surface area of a cylindrical roller, with a diameter of 40 cms and height of 30 cms.

Solution: The curved surface area of a cylinder is given by the following formula,

A = 2.π.r.h

In the question, we are given the diameter of the cylinder,

Thus, the radius,

r = 402

r = 20 cm.

The curved surface area of the cylindrical roller,

A = 2πrh,

A = 2 x π x 20 x 30,

A = 1200 x π,

Taking the value of π as 3.14,

A = 1200 x 3.14,

A = 3768 cm2.

Example 5: Find the curved surface area of a cone with a slant height (l) 12 cm and base diameter (d) of 50 cm.

Solution: In the given question,

Slant height of cone (l) = 12 cm,

Diameter of cone (d) = 50 cm,

Therefore, the base radius of the cone (r) = 502

r = 25 cm

The curved surface area of a cone is given by,

A = π.r.l

Implying the formula with the given values,

Curved surface area = π x 25 x 12,

= π x 300,

Taking the value of π as 3.14,

= 3.14 x 300

= 942 cm2

The curved surface area of the given cone is 942 cm2.

Example 6: Find the area of a hollow sphere with a radius of 7 cm.

Solution: The surface area of a sphere is given by,

$A = 4.\pi.r2.$

In the question, we are given the radius $r = 7$ cm.

Then, the surface area of the sphere,

$A = 4 \times \pi \times (7)2$

Taking the value of π as 22/7,

$A = 4 \times 22/7 \times 7 \times 7,$

$A = 4 \times 22 \times 7$

$A = 616$ cm2

The surface area of the given sphere is 616 cm2.

Example 7: A bowl in the shape of a hemisphere has a diameter of 10 cm; find the bowl's volume.

Solution: The hemispherical bowl has a diameter $d = 10$ cm,

The radius of the bowl $r = 102$

$r = 5$ cm,

The volume of a hemisphere is given by,

$V = 23.\pi\, r3,$

$V= 23.\ \pi\ (5)3$

$V = 23.\pi \times 125,$

Taking the value of π as 3.14

$V = 23 \times 3.14 \times 125,$

$V = 261.67$ cm3.

Therefore, the volume of the hemispherical bowl is 261.67 cm3.

Example 8: Find the volume of a hollow sphere with a radius of 9 cm.

Solution: The volume of a sphere is given by,

$V = 43.\ \pi.r3$

The radius of the given sphere $r = 9$ cm,

The volume of the sphere,

$V = 43.\ \pi \times (9)3,$

$V = 43 \times \pi \times 9 \times 9 \times 9,$

$V = 4 \times \pi \times 3 \times 81,$

Taking the value of π as 3.14,

V = 4 x 3.14 x 3 x 81,

V = 3052.08 cm3

The volume of the given sphere is 3052.08 cm3.

Example 7: A building is standing on a cylindrical pillar of base radius 10 m; if the pillar's height is 10 m, what amount of building material will be needed to make ten more such pillars?

Solution: The radius of the pillar h = 10 m,

Height of the pillar h = 10 m,

the amount of building material used in the pillar,

Volume of the pillar = π.r2.h

= π x 102 x 10,

Taking the value of π as 3.14,

= 3.14 x 1000

The total volume of the pillar,

V = 3140 m3,

The total amount of building material required to make ten such pillars,

Volume of one pillar x 10,

= 3140 x 10,

= 31400 m3

Ten pillars would require 31400 m3 building material.

3.23 Polygons:

Polygons are defined as two-dimensional closed shapes that are formed by joining three or more line segments with each other. We tend to encounter polygons mostly while we learn about geometry. In this lesson, let us learn about polygons definition, regular polygons, polygon sides, and the properties of polygons, along with polygon examples and their identification.

- **What are Polygons?**

The word 'Polygon' is derived from a Greek word in which 'poly' means 'many' and 'gon' means 'angle'. This means that a polygon is a closed figure that is formed by straight lines and these straight lines form the interior angles in it. Polygons can be commonly seen around us. For example, the shape of a honeycomb is a polygon with 6 sides and is known as a hexagon. Each polygon is different in structure and is categorized based on the number of sides and its properties. It should be noted that all polygons are closed plane shapes.

- **Polygon Definition:**

In geometry, the definition of a **polygon** is given as a closed two-dimensional figure which is formed by three or more straight lines.

3.24 Properties of Polygons:

The properties of polygons help us identify them easily. In other words, the following characteristics of a polygon help us to easily check whether a given shape is a polygon or not

- A polygon is a closed shape, that is, there is no end that is left open in the shape. It ends and begins at the same point.
- It is a plane shape, that is, the shape is made of line segments or straight lines.
- It is a two-dimensional figure, that is, it has only two dimensions length and width. There is no depth or height to it.
- It has three or more sides in it.
- The angles in the polygon may or may not be the same.
- The length of the sides of a polygon may or may not be the same.

- **Polygon Sides:**

The sides of a polygon define the name of the specific polygon because different polygons have different number of sides. For example, if a polygon has 3 sides, then it is called a triangle, whereas, if a polygon has 4 sides, it is a quadrilateral. The following section shows the different types of polygons along with their names based on the number of sides.

3.25 Types of Polygons:

There are different types of polygons and they have different names depending on the number of sides that they have. For example, a 3-sided polygon is a triangle, a 4-sided polygon is a quadrilateral, a 5-sided polygon is a pentagon, a 6-sided polygon is a hexagon, and so on.

3.26 Polygon Chart:

The following chart shows the naming convention of polygons on the basis of the number of sides that they have. Each polygon is given a special name on the basis of its number of sides. For example, the trigon, also known as the triangle is made of two words 'tri' which means three, and 'gon' means angles. This shows that it is a shape that has three angles. Observe the table given below to see the names of different polygons as per their number of sides.

Sides	Name of Polygon
3	Trigon (Triangle)
4	Tetragon (Quadrilateral)
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Hendecagon
12	Dodecagon
13	Tridecagon
14	Tetradecagon
15	Pentadecagon
16	Hexadecagon
17	Heptadecagon
18	Octadecagon
19	Enneadecagon
20	Icosagon

3.27 Difference Between Regular and Irregular Polygons:

A polygon can be categorized as a regular or irregular polygon based on the length of its sides and the measure of its angles. The difference between a regular and irregular polygon is given in the following table.

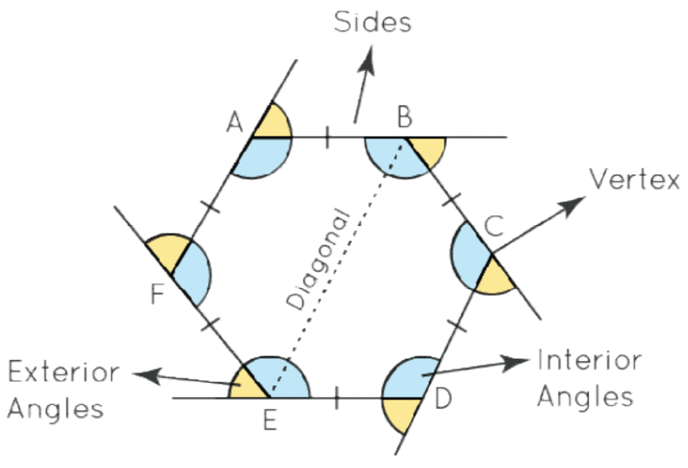
Criterion of Difference	Regular Polygon	Irregular Polygon
Length of sides	Equal	Unequal
Measurement of all interior angles	Equal	Unequal
Measurement of all exterior angles	Equal	Unequal

3.28 Regular Polygons and Irregular Polygons:

It is said that as per Euclidean Geometry, a polygon that is equiangular and equilateral is called a regular polygon while a polygon whose sides are not equiangular and equilateral is referred to as an irregular polygon. Regular polygons are always convex, i.e., all the interior angles measure less than 180º.

In simple words, a regular polygon has all angles of the same measure at each vertex, and all sides of the same length; while a polygon that has sides of different lengths and angles of different measures is referred to as an irregular polygon.

Observe the figure of a regular hexagon given below to understand the parts of a regular polygon.



Here,

- Vertices of the hexagon: A, B, C, D, E, and F
- All the sides of this regular hexagon are equal, i.e., AB = BC = CD = DE = EF = FA
- All the interior angles are equal (represented in blue)
- All the exterior angles are equal (represented in yellow)
- BE is the diagonal.

- **Angles in a Regular Polygon:**

As we learned above, there are two kinds of angles that can be found in the case of a regular polygon. They are:

- Interior Angles of a Polygon
- Exterior Angles of a Polygon

- **Interior Angles of a Polygon:**

The interior angles are formed between the adjacent sides inside the polygon and are equal to each other in the case of a regular polygon. The number of interior angles is equal to the number of sides. The value of an interior angle of a regular polygon can be calculated if the number of sides of the regular polygon is known by using the following formula:

Interior angle = $180^\circ(n-2)/n$, where 'n' is the number of sides

- **Exterior Angles of a Polygon:**

Each exterior angle of a regular polygon is formed by extending one side of the polygon (either clockwise or anticlockwise) and then the angle between that extension and the adjacent side is measured. Each exterior angle of a regular polygon is equal and the sum of the exterior angles of a polygon is 360°. An exterior angle can be calculated if the number of sides of a regular polygon is known by using the following formula:

Exterior Angle = $360^\circ/n$, where 'n' is the number of sides of the polygon

- **Important Notes on Polygons:**

- Polygons are 2-D figures with more than 3 sides.
- The angles of a regular polygon can be measured by using the following formulas: Exterior Angle = $360^\circ/n$
  Interior angle = $180^\circ(n-2)/n$, where n refers to the number of sides.
- The sum of interior and exterior angles at a point is always 180º as they form a linear pair of angles
- For an 'n'-sided polygon, the number of diagonals can be calculated with this formula, $n(n-3)/2$

- **Polygon Formulas:**

There are two basic formulas for polygons listed below:

- Area of polygons
- Perimeter of polygons

Let us learn about the above-listed two polygon formulas in detail.

- **Area of Polygons:**

The area of a polygon is defined as the measurement of space enclosed within a polygon. The area of polygons can be found by different formulas depending upon whether the polygon is a regular or an irregular polygon. For example, a triangle is a three-sided polygon which is known as a trigon. The formula for calculating the area of the trigon (triangle) is half the product of the base and height of the triangle. It is expressed in square units like, m^2, cm^2, ft^2. Similarly, each polygon has a different formula depending on the number of sides and the type of polygon.

- **Perimeter of Polygons:**

The perimeter of a polygon is defined as the distance around a polygon which can be obtained by summing up the length of all given sides.

The perimeter of polygon formula = Length of Side 1 + Length of Side 2 + Length of Side 3...+ Length of side N (for an N-sided polygon). It is expressed in terms of units such as meters, cm, feet, etc.

3.29 Concave and Convex Polygons:

Concave polygons are those polygons that have at least one interior angle which is a reflex angle and it points inwards. Concave polygons have a minimum of 4 sides and a few of the diagonals in a concave polygon may lie partly or fully outside it. It is to be noted that all concave polygons are irregular because the interior angles are not equal.

On the other hand, a convex polygon has no interior angle that measures more than 180°. A convex polygon can have 3 sides and no diagonal in a convex polygon lies outside it.

3.30 What is a true discount?

True Discount is a concept that defines the difference between the amount due and the present value of the amount. In other ways, it can be said that a True discount is nothing but the interest on the present amount of money. In this field, it is important to clarify the idea of present discount and the Banker's discount. Present worth is the concept of money that is to be paid before the upcoming date or the due date. On the other hand, Banker's discount is called the simple interest on the due amount or on the face value and in this field, the bill was discounted.

- **The formula of true discount:**

The difference between the present value of the amount and the due an\mount is called true discount and it has a formula. The formula is:

PV* (1+rt) = FV

In this field FV is the face value, r is simple interest, and t stands for time. It can be said that in this case of the study true discount is the difference between the face value and true value. An example of it can be given as two numbers are 1050 and 1.025 and in this field 1050 is the face value, and 1.025 is the true value. Therefore, the true value for it is 1024.4 and this is the important factor of true discount.

3.31 Discussion on True discount:

The true discount is a concept that defines many factors of present worth and therefore it is important to clarify the concept of present worth in this field. Any kind of money that has to be paid before the date of due is cleared off and it is for debt is known as the present birth of the money. True discounts can be discussed after the discussion of present worth. True discount contains a formula and it is an important part of this present discount.

- **Problems of true discount:**

It can be seen that true discount is a concept that describes the difference between the due amount and the present value of the worth of this amount. This factor of true discount has some problems in the calculator if it. They are-

- The calculation is tough in nature
- Needs proper mathematical calculation
- Tough to understand

Having these problems it can be said that true discount is a process that is tough to understand and the nature of it is also tough. The important factor of true discount is that it is also called the interest on the present worth.

3.32 Banker's discount and True discount:

A banker's discount is called the simple interest on the face value and it is on the debt on the period. It goes from the date on which the bill is discontinued to the legal due date. The interest is deducted by the Banker on the face value and it is for the unexpired time. Banker's discount has the formula = FV*r*t

True discount in this aspect is the simple interest on the present value or it can also be called worth for the unexpired time. It has the formula, True discount = Face value – Present Value.

3.33 Difference between Simple Interest and True Discount:

The amount of interest that is saved after the payment of present worth is described as the true discount. It can also be seen as the difference between the total amount and the present worth. In this field, the amount is the value after the interest is added to the present value. Therefore the idea can be drawn that the concept of true discount is the same as simple interest.

3.34 Banker's Discount:

Banker's discount is a type of interest charged on a bill of exchange that is discounted before it is due. It is a way for banks to earn income on their loans.

Here is a note on banker's discount:

Banker's discount is calculated on the face value of the bill, the rate of interest, and the time to maturity. The face value of the bill is the amount that is due on the due date. The rate of interest is the interest rate that the bank charges. The time to maturity is the number of days until the bill is due.

The formula for banker's discount is:

Bankers discount = Face value of the bill x Rate of interest x Time to maturity / 365

For example, suppose a bill of exchange has a face value of ₹1,000, a rate of interest of 5%, and is due in 60 days. The bankers discount would be:

Bankers discount = ₹1,000 x 5% x 60 days / 365 = ₹8.22

The banker would deduct the bankers discount from the face value of the bill to determine the amount that the borrower would receive. In this example, the borrower would receive ₹991.78.

Banker's discount is typically used for short-term loans. It is a relatively expensive form of financing, but it can be a good option for borrowers who need cash quickly.

Examples and Solutions:

Example 1: A bill of exchange has a face value of ₹10,000, a rate of interest of 6%, and is due in 90 days. What is the banker's discount?

Solution:

Banker's discount = Face value of the bill x Rate of interest x Time to maturity / 365 Bankers discount = ₹10,000 x 6% x 90 days / 365 = ₹148.34

Answer: The banker's discount is ₹148.34.

Example 2: A bill of exchange has a face value of ₹5,000 and is due in 60 days. The bank discounts the bill at a rate of 7%. What is the amount that the borrower will receive?

Solution:

Banker's discount = Face value of the bill x Rate of interest x Time to maturity / 365 Banker's discount = ₹5,000 x 7% x 60 days / 365 = ₹57.53

Amount that the borrower will receive = Face value of the bill – Banker's discount Amount that the borrower will receive = ₹5,000 - ₹57.53 = ₹4,942.47

Answer: The amount that the borrower will receive is ₹4,942.47.

3.35 Calendar:

A calendar is a system of organizing time that divides a year into days, weeks, and months. Calendars are used to keep track of events, appointments, and deadlines.

There are many different types of calendars, but the most common is the Gregorian calendar. The Gregorian calendar is a solar calendar that is based on the Earth's orbit around the sun. It was introduced by Pope Gregory XIII in 1582 to replace the Julian calendar, which was inaccurate.

The Gregorian calendar has 365 days in a year, with an extra day added every fourth year to make up for the fact that the Earth's orbit around the sun is not exactly 365 days long. The extra day is called a leap day, and it is usually February 29th.

The Gregorian calendar is used by most of the world today, but there are still some countries that use other calendars. Some examples of other calendars include the Islamic calendar, the Hebrew calendar, and the Chinese calendar.

Also, there are several Indian calendars, the most prominent being the Hindu calendar. It's a lunisolar calendar with various regional variations, used for religious and social purposes in the Indian subcontinent and Southeast Asia.

Here are some key points about the Hindu calendar:

- **Lunar and Solar Components:** It incorporates both lunar (Chandramana) and solar (Sauramana) cycles.
- **Variants:** There are multiple variants, including the Vikrama Samvat (used in North India) and the Shaka Samvat (used in South India).
- **Religious Significance:** It's closely tied to Hindu festivals and rituals, determining auspicious dates for ceremonies.
- **Timekeeping:** It uses a unique system of timekeeping, dividing the day into smaller units like Muhurta and Ghatika.

Example 1: Leap Year
A leap year is a year with an extra day, February 29th. This is added to keep the calendar year synchronized with the solar year. A leap year occurs every 4 years, except for years divisible by 100 but not by 400.

Example 2: Time Zones
Different parts of the world observe different time zones. This is necessary to adjust for the Earth's rotation. For example, India Standard Time (IST) is 5 hours and 30 minutes ahead of UTC (Universal Time Coordinated).

- **Here are some of the key features of a calendar:**
  - Days: Days are the smallest unit of time in a calendar. They are typically divided into 24 hours.
  - Weeks: Weeks are a group of seven days. The first day of the week varies depending on the country and culture.
  - Months: Months are a group of four or five weeks. There are twelve months in a year.
  - Years: Years are a period of 365 days. A year is divided into four seasons: spring, summer, autumn, and winter.
  - Leap years: Leap years are years that have an extra day, February 29th. Leap years occur every fourth year, except for years that are divisible by 100 but not by 400.
- **Calendars can be used for a variety of purposes, including:**
  - Keeping track of appointments and deadlines
  - Planning events
  - Tracking personal finances
  - Keeping track of important dates, such as birthdays and anniversaries
  - Learning about history and culture

There are many different ways to use a calendar. Some people prefer to use paper calendars, while others prefer to use digital calendars. There are also many different calendar apps available for smartphones and tablets.

Examples and Solutions:

Example 1: What is the significance of the Hindu calendar in Indian culture?

Solution: The Hindu calendar plays a crucial role in Indian culture. It determines the auspicious dates for various religious festivals, ceremonies, and rituals. It also influences social and agricultural practices, as many festivals are tied to specific seasons and astronomical events.

Example 2: What are the main components of the Hindu calendar?

Solution: The Hindu calendar primarily consists of two components:

1. **Lunar Calendar (Chandramana):** Based on the lunar cycles, it determines the duration of months and is crucial for religious observances.
2. **Solar Calendar (Sauramana):** Based on the solar cycle, it determines the seasons and is used for agricultural purposes.

By combining these two components, the Hindu calendar provides a comprehensive framework for timekeeping and cultural practices.

Example 3: If today is Tuesday, November 5, 2024, what day of the week will it be 30 days from now?

Solution: To solve this, we need to consider the number of weeks in 30 days.

i)30 days ÷ 7 days/week ≈ 4 weeks and 2 days.

So, after 4 weeks, it will again be Tuesday. Then, adding 2 more days, we get:

ii) Tuesday + 2 days = Thursday

Therefore, 30 days from November 5, 2024, it will be Thursday, December 5, 2024.

Example 4: If a particular Hindu year starts on the 15th of March, 2024, and the Hindu year is composed of 12 months, each of 30 days, on what date will the 7th month of that year begin?

Solution: 7 months * 30 days/month = 210 days So, the 7th month will start on 15th March + 210 days = 12th November, 2024.

3.36 Clock:

A clock is a device used to measure and indicate time. It typically has a dial with hour and minute hands (and sometimes a second hand) that rotate around a central point.

3.37 Types of Clocks:

1. **Analog Clocks:** These clocks display time using hands that move around a circular dial.
2. **Digital Clocks:** These clocks display time using numbers on a screen.
3. **Atomic Clocks:** Extremely precise clocks that use the vibrations of atoms to measure time.

3.38 Time Measurement:

- **Hours:** A standard unit of time typically divided into 24 hours in a day.
- **Minutes:** A smaller unit of time, 60 minutes make up an hour.
- **Seconds:** The smallest unit of time commonly used; 60 seconds make up a minute.

- **Time Zones:**

Different parts of the world observe different time zones. This is necessary to adjust for the Earth's rotation.

- **Clock Problems and Puzzles:**

Many puzzles and riddles involve clocks, testing one's understanding of time and angles. For example:

- **Angle Between Hands:** Calculating the angle between the hour and minute hands at a specific time.
- **Time After/Before:** Determining the time after or before a given time, considering the movement of clock hands.
- **Mirror Image Time:** Finding the mirror image of a time displayed on a clock.

Examples and Solutions:

Example 1: At what time between 3 PM and 4 PM, are the hour and minute hands of a clock 15 minutes apart?

Solution: Let's assume the time is x minutes past 3 PM. The hour hand moves 360 degrees in 12 hours, or 0.5 degrees per minute. So, in x minutes, it moves 0.5x degrees past the 3 o'clock mark.

The minute hand moves 360 degrees in 60 minutes, or 6 degrees per minute. So, in x minutes, it moves 6x degrees.

The angle between the hands should be 15 minutes, which is $15/60 * 360 = 90$ degrees.

Therefore, we can set up the equation: $6x - 0.5x = 90$ $5.5x = 90$ $x \approx 16.36$ minutes

So, the time is approximately 16.36 minutes past 3 PM.

Example 2: A clock gains 10 minutes every hour. If it is set right at 5 AM, what time will it show at 11 AM on the same day?

Solution: From 5 AM to 11 AM, there are 6 hours. In 1 hour, the clock gains 10 minutes. So, in 6 hours, it gains $6 * 10 = 60$ minutes = 1 hour.

Therefore, the clock will show 11 AM + 1 hour = 12 PM.

3.39 Pie Chart:

A pie chart is a circular statistical graphic that represents numerical proportions by dividing a circle (pie) into proportional slices. It's a visual representation of data where each slice of the pie represents a category, and the size of the slice corresponds to the proportion of the category to the whole.

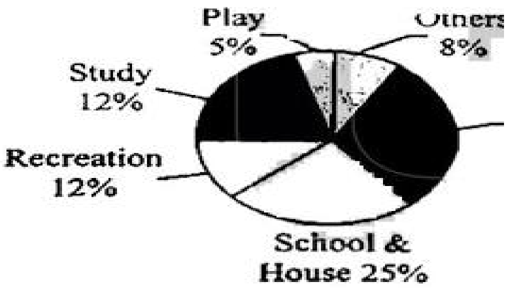
Example:

Let's say we have data on how a student spends their 24-hour day:

- **Sleep:** 8 hours
- **School:** 7 hours
- **Homework:** 2 hours
- **Play:** 5 hours
- **Other activities:** 2 hours

Steps to create a pie chart:

1. **Calculate the percentage of each category:**
   - Sleep: (8/24) * 100% = 33.33%
   - School: (7/24) * 100% = 29.17%
   - Homework: (2/24) * 100% = 8.33%
   - Play: (5/24) * 100% = 20.83%
   - Other activities: (2/24) * 100% = 8.33%
2. **Calculate the angle for each category:**
   - Sleep: 33.33% * 360° = 120°
   - School: 29.17% * 360° = 105°
   - Homework: 8.33% * 360° = 30°
   - Play: 20.83% * 360° = 75°
   - Other activities: 8.33% * 360° = 30°
3. **Draw a circle and divide it into sectors based on the calculated angles.**



(Fig.pie chart showing the student's daily activities)

- **Interpreting the Pie Chart:**
  - The largest slice, "Sleep," indicates that the student spends the most time sleeping.
  - The smallest slice, "Homework," shows that the student spends the least amount of time on homework.
  - The pie chart visually represents the distribution of the student's time across different activities.

Pie charts are useful for visualizing categorical data and understanding the relative proportions of different categories within a whole. However, they can become less effective when there are too many categories or when the differences between categories are subtle.

Examples and Solutions:

Example 1: If a pie chart represents the expenses of a family, and the sector for food expenses is 40°, what percentage of the total expenses is spent on food?

Solution: A full circle is 360°. If 360° represents 100% of the expenses, then 40° represents (40/360) * 100% = 11.11% of the expenses.

Example 2: In a pie chart, the angle of the sector representing the number of students who passed an exam is 144°. If the total number of students who appeared for the exam is 250, how many students passed the exam?

Solution: 144° out of 360° represents the fraction (144/360) of the total students. So, the number of students who passed is (144/360) * 250 = 100 students.

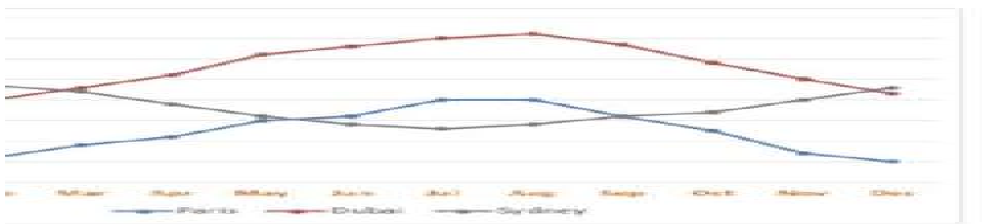
3.40 Line Graph:

A line graph is a type of chart used to visualize data points connected by straight lines. It's particularly useful for showing trends over time, such as temperature changes, stock prices, or population growth.

- **Key Components of a Line Graph:**
  - **X-axis:** Represents the independent variable (e.g., time, date, or category).
  - **Y-axis:** Represents the dependent variable (e.g., temperature, sales, or population).
  - **Data Points:** Individual data points plotted on the graph.
  - **Line Segments:** Lines connecting the data points to show trends and patterns.

Example:

Let's consider a line graph showing the average monthly temperature in a city over a year:



(Fig.line graph showing average monthly temperature)

- **Interpreting the Line Graph:**
  - **Trends:** The line's direction indicates the trend. An upward slope shows an increase, a downward slope shows a decrease, and a horizontal line indicates no change.
  - **Peaks and Valleys:** High points on the line represent peaks, while low points represent valleys.
  - **Rate of Change:** The steepness of the line indicates the rate of change. A steeper line shows a faster rate of change.

Examples and Solutions:

Example 1: Based on the line graph, in which month was the highest average temperature recorded?

Solution: By analysing the peaks on the graph, we can identify the month with the highest point. In this example, it appears to be the month of July.

Example 2: Between which two months was the most significant decrease in average temperature observed?

Solution: To determine the most significant decrease, we need to look for the steepest downward slope on the graph. This would indicate a rapid decline in temperature. In this example, it might be between the months of October and November.

Example 3: Using the data from the line graph, answer the following questions:

1. **What was the highest temperature recorded during the week, and on which day was it recorded?**
2. **Calculate the average temperature for the week.**
3. **What was the difference in temperature between the hottest day and the coolest day?**
4. **How many days had a temperature above the weekly average? List these days.**
5. **If the trend continued, estimate the temperature on the following Monday if the temperature is expected to rise by 2°C.**

Solution:

1. Highest temperature recorded during the week and the day it was recorded:

From the table, we can see that the highest temperature recorded was **28°C on Saturday**.

2. Average temperature for the week:

To find the average temperature, add up the temperatures for each day and divide by the number of days (7).

Total Temperature=22+24+21+23+26+28+25=169 °C

Average Temperature=1697≈24.14 °C

So, the **average temperature for the week was approximately 24.14°C**.

3. Difference in temperature between the hottest day and the coolest day:

The hottest temperature was **28°C** (Saturday), and the coolest temperature was **21°C** (Wednesday).

Temperature Difference=28−21=7 °C

So, the **difference in temperature between the hottest and coolest days was 7°C**.

4. Days with temperatures above the weekly average:

The weekly average temperature is approximately 24.14°C. Now, list the days where the temperature was above this average.

- **Tuesday:** 24°C (slightly below average)
- **Friday:** 26°C (above average)
- **Saturday:** 28°C (above average)
- **Sunday:** 25°C (above average)

So, **the days with temperatures above the weekly average were Friday, Saturday, and Sunday**.

5. Estimated temperature for the following Monday if the trend continues with a 2°C increase:

If the temperature is expected to rise by 2°C, we can estimate the temperature for the following Monday.

Temperature on previous Sunday=25 °C Estimated Temperature on following Monday=25+2=27 °C

So, the **estimated temperature for the following Monday would be 27°C** if the trend of a 2°C increase continues.

Example 4: Based on the data provided in the line graph, answer the following:

1. **Calculate the total revenue for each company over the six-month period. Which company generated the highest revenue overall?**
2. **Determine the percentage increase in revenue from January to June for each company. Which company had a higher growth rate?**
3. **Calculate the average monthly revenue for each company.**
4. **In which month did the revenue difference between the two companies reach its maximum, and what was the difference?**
5. **If the trend continues, project the revenue for each company in July, assuming each company's revenue grows by the average month-over-month growth rate from January to June.**

Solution:

1. Total revenue for each company over the six-month period:

To find the total revenue, sum the monthly revenues for each company.

- **Company A:** 50+55+60+75+80+95=415 (thousands)
- **Company B:** 40+45+50+60+70+80=345 (thousands)

So, **Company A generated the highest revenue overall**, with **₹415,000** over six months.

2. Percentage increase in revenue from January to June for each company:

To calculate the percentage increase in revenue from January to June:

Percentage Increase=(Revenue in June−Revenue in January) ×100/Revenue in January

For **Company A**:=(95−50) ×100/50=90%

For **Company B**:=(80−40)×100/40=100%

Company B had a higher growth rate, with a 100% increase in revenue from January to June, compared to Company A's 90% increase.

3. Average monthly revenue for each company:

To find the average monthly revenue, divide the total revenue by 6 (the number of months).

- **Company A:** 415/6≈69.17 (thousands)
- **Company B:** 345/6≈57.5 (thousands)

So, the **average monthly revenue for Company A was approximately₹69,170**, while **Company B's average monthly revenue was approximately ₹57,500**.

4. Month with the maximum revenue difference between the two companies:

To find the month with the largest difference, calculate the monthly revenue difference between Company A and Company B.

- **January:** 50−40=10 (thousands)
- **February:** 55−45=10(thousands)
- **March:** 60−50=10 (thousands)
- **April:** 75−60=15(thousands)
- **May:** 80−70=10 (thousands)
- **June:** 95−80=15 (thousands)

The revenue difference was **greatest in April and June**, with a difference of ₹**15,000**.

5. Projected revenue for each company in July:

To project the revenue for July, we first calculate the average month-over-month growth rate for each company from January to June, then apply this growth rate to June's revenue.

Step 1: Calculate the total growth over 5 intervals (from January to June).

For **Company A**:

Total Growth=95−50=45 (thousands)

Average Monthly Growth for Company A=45/5=9 (thousands)

For **Company B**:

Total Growth=80−40=40 (thousands) Average Monthly Growth for Company B=40/5=8 (thousands)

Step 2: Add the average monthly growth to June's revenue to project July's revenue.

For **Company A**:

Projected Revenue in July=95+9=104 (thousands)

For **Company B**: {Projected Revenue in July} = 80 + 8 = 88 (thousands)

So, **if the trend continues, Company A's projected revenue for July is ₹104,000, and Company B's projected revenue is ₹88,000**.

3.41 Bar Graph:

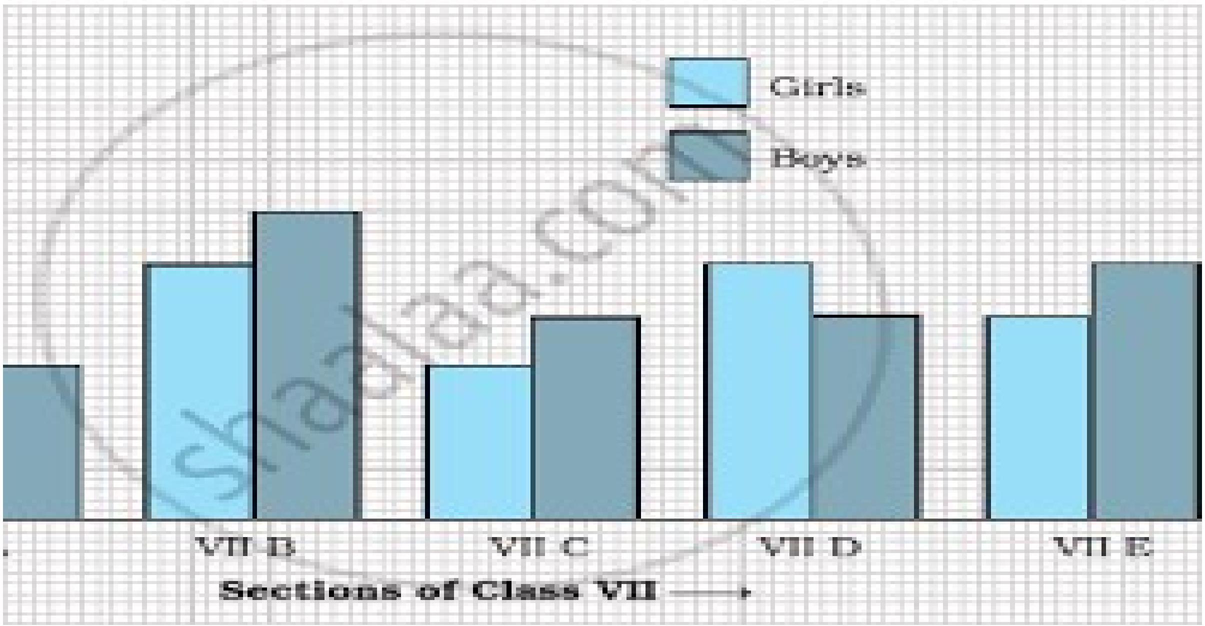
A bar graph is a chart that uses rectangular bars to represent data. It's a visual tool to compare different categories or groups. Bar graphs can be vertical or horizontal.

3.42 Types of Bar Graphs:

1. **Simple Bar Graph:** Used to compare different categories.
2. **Compound Bar Graph:** Used to compare multiple variables within a category.
3. **Stacked Bar Graph:** Used to show the composition of a category.

Example:

Let's consider a bar graph showing the number of students in different sections of a class:



(Fig. bar graph showing the number of students in different sections)

- **Interpreting the Bar Graph:**
  - **Comparison:** The height of each bar represents the value of the corresponding category.
  - **Ranking:** The bars can be compared to rank categories based on their values.
  - **Total Value:** The total value can be estimated by adding up the heights of all the bars.

Examples and Solutions:

Example 1: Which section has the highest number of students?

Solution: By comparing the heights of the bars, we can see that Section A has the highest number of students.

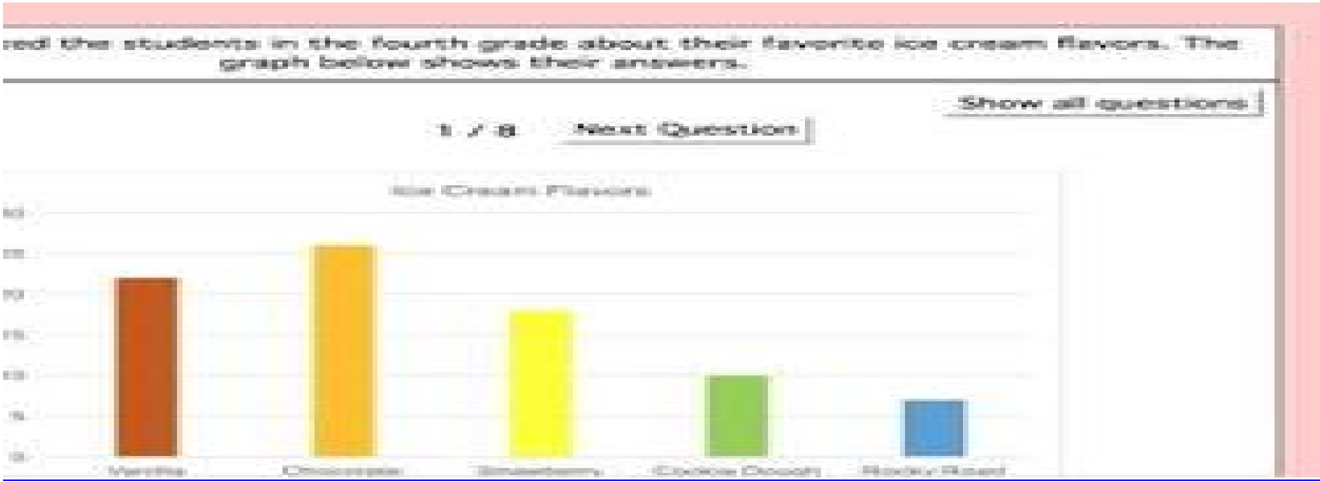
Example 2: What is the total number of students in all sections combined?

Solution: To find the total number of students, we need to add the values represented by each bar. For example, if the heights of the bars represent the number of students in each section, we would add the heights of all the bars to get the total.

Example 3: A survey was conducted to determine the favourite ice cream flavours among a group of people. The results are as follows:

- **Chocolate:** 30%
- **Vanilla:** 25%
- **Strawberry:** 20%
- **Mint Chocolate Chip:** 15%
- **Cookies and Cream:** 10%

Bar Graph:



(Fig.bar graph showing the favourite ice cream flavours)

➢ Which flavour is the most popular?

Solution: By observing the bar graph, we can see that the bar for **Chocolate** is the tallest. Therefore, Chocolate is the most popular flavour.

➢ What percentage of people prefer either Vanilla or Strawberry?

Solution: To find the total percentage for Vanilla and Strawberry, we add their individual percentages:

25% (Vanilla) + 20% (Strawberry) = 45%

So, 45% of people prefer either Vanilla or Strawberry.

Example 4: If a total of 100 people participated in the survey, how many people preferred Mint Chocolate Chip?

Solution: Since 15% of the people preferred Mint Chocolate Chip, and the total number of people is 100, we can calculate the number of people:

- 15% of 100 = (15/100) * 100 = 15 people.

Therefore, 15 people preferred Mint Chocolate Chip.

OBJECTIVE TYPE QUESTIONS (EXERCISE)

BASED ON UNIT-I, II AND III

Q1. The HCF of 36 and 60 is:
a) 6
b) 12
c) 18
d) 24

Q2. The LCM of 15 and 25 is:
a) 25
b) 50
c) 75
d) 100

Q3. If the HCF of two numbers is 7 and their LCM is 210, and one of the numbers is 35, what is the other number?
a) 14
b) 15
c) 21
d) 30

Q4. The LCM of two numbers is 180, and their HCF is 6. If one number is 36, what is the other number?
a) 30
b) 45
c) 50
d) 60

Q5. Which of the following is a terminating decimal?
a) 1/7
b) 3/8
c) 5/3
d) 7/6

Q6. Convert 0.375 to a fraction.
a) 3/8
b) 5/16
c) 7/20
d) 2/5

Q7. Which of the following is the correct decimal form of 58\frac{5}{8}85?
a) 0.625
b) 0.75
c) 1.25
d) 0.85

Q8. Simplify: $15\times2/5$
a) 3
b) 6
c) 5
d) 10

Q9. Simplify the expression:
$18 \div 3 \times 4 + 2$
a) 24
b) 20
c) 28
d) 18

Q10. The square root of 144 is:
a) 12
b) 14
c) 16
d) 18

Q11. What is the cube root of 27?
a) 2
b) 3
c) 4
d) 5

Q12. The square root of 625 is:
a) 25
b) 20
c) 15
d) 30

Q13. What is the cube root of 64?
a) 2
b) 4
c) 6
d) 8

Q14. The sum of the ages of A and B is 50 years. Ten years ago, the ratio of their ages was 3:4. What is the present age of A?
a) 20 years
b) 30 years
c) 25 years
d) 35 years

Q15. A man is 24 years older than his son. In 6 years, the father's age will be twice the son's age. What are their present ages?
a) Father: 30, Son: 6
b) Father: 36, Son: 12
c) Father: 42, Son: 18
d) Father: 48, Son: 24

Q16. A father’s age is three times the age of his son. In 15 years, the father will be twice as old as his son. How old is the father now?
a) 30 years
b) 36 years

c) 45 years
d) 48 years

Q17. The present age of a father is 6 times that of his son. After 12 years, the father's age will be 3 times the age of his son. What is the present age of the father?
a) 48 years
b) 54 years
c) 60 years
d) 72 years

Q18. The sum of the present ages of A and B is 60 years. In 6 years, the ratio of their ages will be 5:7. What is the present age of A?
a) 24 years
b) 28 years
c) 30 years
d) 32 years

Q19. A person buys a book for ₹120 and sells it for ₹150. The percentage profit is:
a) 15%
b) 20%
c) 25%
d) 30%

Q20. If the price of an article increases by 10%, then after a decrease of 10%, the final price of the article is:
a) Same as the original price
b) 1% less than the original price
c) 2% more than the original price
d) 5% more than the original price

Q21. A man buys an article for ₹500 and sells it at a loss of 20%. What is the selling price?
a) ₹380
b) ₹400
c) ₹420
d) ₹450

Q22. A trader marks his goods 25% above the cost price and allows a discount of 10%. What is the profit percentage?
a) 10%
b) 15%
c) 12.5%
d) 18%

Q23. If the cost price of 10 articles is ₹800 and the selling price of 12 articles is ₹1080, what is the profit percentage?
a) 20%
b) 25%
c) 30%
d) 15%

Q24. The ratio of two numbers is 5:8. If the first number is 25, what is the second number?
a) 30
b) 35
c) 40
d) 45

Q25. A mixture contains milk and water in the ratio 3:5. If the total quantity of the mixture is 40 liters, how much milk is in the mixture?
a) 15 liters
b) 20 liters
c) 24 liters
d) 25 liters

Q26. If the ratio of the ages of A and B is 4:5 and the sum of their ages is 72, what is the age of B?
a) 32 years
b) 36 years
c) 40 years
d) 44 years

Q27. A car travels 120 km in 3 hours. What is its speed?
a) 30 km/h
b) 40 km/h
c) 50 km/h
d) 60 km/h

Q28. A man can complete a work in 15 days. How long will it take for 5 men to complete the same work?
a) 3 days
b) 5 days
c) 7 days
d) 15 days

Q29. A train 150 meters long is running at a speed of 72 km/h. How long will it take to cross a bridge 200 meters long?
a) 12 seconds
b) 15 seconds
c) 18 seconds
d) 20 second

Q30. Find the simple interest on ₹1000 at the rate of 5% per annum for 2 years.
a) ₹50
b) ₹100
c) ₹150
d) ₹200

Q31. If the principal is ₹1500, rate of interest is 6% per annum, and the time is 3 years, then the simple interest is:
a) ₹270
b) ₹300

c) ₹350
d) ₹400

Q32. A sum of money amounts to ₹2200 in 2 years at 10% simple interest per annum. What is the principal amount?
a) ₹1800
b) ₹2000
c) ₹1500
d) ₹1600

Q33. What is the compound interest on ₹1500 at 10% per annum for 2 years, compounded annually?
a) ₹300
b) ₹315
c) ₹330
d) ₹350

Q34. If the compound interest on a certain sum of money is ₹800 at the rate of 5% per annum for 2 years, what is the principal amount?
a) ₹8000
b) ₹6000
c) ₹5000
d) ₹4000

Q35. A sum of money doubles itself in 5 years at 10% compound interest per annum. How long will it take for the sum to triple?
a) 8 years
b) 10 years
c) 12 years
d) 15 years

Q36.What is the area of a circle with a radius of 7 cm?
A) 49π cm^2
B) 14π cm^2
C) 7π cm^2
D) 49 cm^2

Q37.What is the surface area of a cube with a side length of 4 cm?
A) 16 cm^2
B) 64 cm^2
C) 96 cm^2
D) 48 cm^2

Q38.What is the volume of a cylinder with a radius of 5 cm and height of 10 cm? (Use $\pi = 3.14$)
A) 785 cm^3
B) 1570 cm^3
C) 314 cm^3
D) 250 cm^3

Q39.The volume of a cone is given by the formula V=13πr2hV = \frac{1}{3}πr^2hV=31πr2h. What is the volume of a cone with a radius of 3 cm and height of 9 cm? (Use π = 3.14)

A) 84.78 cm^3
B) 85.75 cm^3
C) 84.88 cm^3
D) 85.50 cm^3

Q40.What is the surface area of a sphere with a radius of 6 cm? (Use π = 3.14)

A) 113.04 cm^2
B) 226.08 cm^2
C) 75.36 cm^2
D) 113.16 cm

Q41.What is the area of a triangle with a base of 8 cm and height of 5 cm?

A) 30 cm^2
B) 20 cm^2
C) 40 cm^2
D) 25 cm^2

Q42.What is the volume of a rectangular prism with dimensions 3 cm by 4 cm by 5 cm?

A) 60 cm^3
B) 120 cm^3
C) 150 cm^3
D) 70 cm^3

Q43.What is the surface area of a rectangular prism with length 4 cm, width 3 cm, and height 2 cm?

A) 52 cm^2
B) 48 cm^2
C) 24 cm^2
D) 34 cm^2

Q44.What is the surface area of a right circular cylinder with radius 4 cm and height 10 cm? (Use π = 3.14)

A) 240.56 cm^2
B) 251.2 cm^2
C) 257.6 cm^2
D) 307.2 cm^2

Q45.What is the volume of a sphere with radius 4 cm? (Use π = 3.14)

A) 268.08 cm^3
B) 268.1 cm^3

C) 268 cm^3
D) 256 cm^3

Q46.If the true discount on a sum of money due in 2 years at 10% simple interest is ₹100, what is the sum?
A) ₹1000
B) ₹1100
C) ₹1200
D) ₹1500

Q47.The true discount on a sum of money due in 3 years at 5% simple interest is ₹225. What is the sum?
A) ₹2250
B) ₹2500
C) ₹3000
D) ₹3500

Q48.A person is to pay ₹1600 after 4 years at 12% simple interest. What is the true discount on the amount?
A) ₹300
B) ₹400
C) ₹500
D) ₹600

Q49.If the sum of money is ₹5000, the true discount is ₹400, and the rate of interest is 5% per annum. What is the time period in years?
A) 4 years
B) 5 years
C) 6 years
D) 3 year

Q50.A sum of ₹4000 is due in 2 years at 6% per annum simple interest. What is the true discount on this sum?
A) ₹220
B) ₹240
C) ₹250
D) ₹260

Q51.A regular polygon has 9 sides. What is the sum of the interior angles of this polygon?
A) 1260°
B) 1440°
C) 144°
D) 1600°

Q52.What is the sum of the interior angles of a convex polygon with 12 sides?
A) 1800°
B) 1440°
C) 1320°
D) 720°

Q53.What is the measure of each exterior angle of a regular polygon with 10 sides?
A) 36°
B) 72°
C) 90°
D) 120°

Q54.Which of the following is not a type of polygon?
A) Hexagon
B) Decagon
C) Nonagon
D) Cylinder

Q55.The area of a regular hexagon is $54\sqrt{3}$ square units. What is the length of one side of the hexagon?
A) 6 units
B) 9 units
C) 12 units
D) 3 units

Q56.A shirt is originally priced at ₹800, but it is available at a 25% discount. What is the price of the shirt after the discount?
A) ₹600
B) ₹575
C) ₹650
D) ₹700

Q57.A product is marked at ₹2000. A 20% discount is first applied, followed by a 10% discount on the reduced price. What is the final price of the product?
A) ₹1440
B) ₹1560
C) ₹1600
D) ₹1800

Q58.A retailer buys a watch for ₹1200 and offers a 20% discount on the marked price. If the watch is sold at ₹1200, what is the profit or loss made by the retailer?

A) 10% Profit
B) 10% Loss
C) 20% Profit
D) 20% Loss

Q59.A product is originally priced at ₹600 and is sold for ₹480. What is the discount percentage?
A) 15%
B) 20%
C) 25%
D) 30%

Q60.A product is priced at ₹5000. It is first given a 15% discount and then a second discount of 10%. What is the final price of the product?
A) ₹3900
B) ₹4250
C) ₹4200
D) ₹4350

Q61.What day of the week will it be on 15th August 2025?
A) Friday
B) Saturday
C) Sunday
D) Wednesday

Q62.Which of the following years is a leap year?
A) 1900
B) 2000
C) 2100
D) 2200

Q63.How many odd days are there in 400 years?
A) 0
B) 1
C) 2
D) 3

Q64.If a year starts on a Monday, what day of the week will it be on 31st December of that year?
A) Monday
B) Tuesday
C) Wednesday
D) Thursday

Q65.What is the day of the week on 1st January 2000?
A) Saturday
B) Sunday
C) Friday
D) Thursday

Q66.At 3:00, what is the angle between the hour and minute hands of the clock?
A) 0°
B) 15°
C) 90°
D) 180°

Q67.At what time do the minute and hour hands of a clock next coincide after 12:00?
A) 12:05
B) 12:10
C) 12:15
D) 12:12

Q68.At 6:30, what is the angle between the hour and minute hands of the clock?
A) 90°
B) 120°
C) 135°
D) 180°

Q69.How many times do the minute and hour hands coincide in a 24-hour period?
A) 22
B) 24
C) 20
D) 11

Q70.If the hands of a clock coincide at 12:00, what is the time between the second and third coincidences?
A) 65 minutes
B) 66 minutes
C) 72 minutes
D) 75 minutes

Q71.What is the angle between the hour and minute hands of the clock at 8:00?
A) 30°
B) 45°
C) 60°
D) 120°

Q72.How much time does the hour hand take to move 30°?
A) 1 hour
B) 30 minutes
C) 2 hours
D) 1.5 hours

Q73.At 9:15, what is the angle between the hour and minute hands?
A) 90°
B) 97.5°
C) 105°
D) 110°

Q74.How many divisions are there on the face of a standard analog clock?
A) 12

B) 24
C) 60
D) 120

Q75.How much time does the minute hand take to move from 12 to 6 on a clock?
A) 30 minutes
B) 25 minutes
C) 45 minutes
D) 60 minutes

Q76.A train 200 meters long passes a station platform in 30 seconds at a speed of 72 km/h. What is the length of the platform?
A) 180 meters
B) 220 meters
C) 250 meters
D) 300 meters

Q77.Two trains, each 100 meters long, are moving in opposite directions at speeds of 60 km/h and 80 km/h. How long will it take for the trains to completely pass each other?
A) 6 seconds
B) 10 seconds
C) 12 seconds
D) 15 seconds

Q78.A train passes a person running at 10 km/h in the same direction. If the train's speed is 50 km/h and the person is 2 meters tall, how long does it take for the train to pass the person completely?
A) 4 seconds
B) 5 seconds
C) 6 seconds
D) 8 seconds

Q79.A train 150 meters long takes 15 seconds to cross a platform. If the speed of the train is 54 km/h, what is the length of the platform?
A) 150 meters
B) 180 meters
C) 200 meters
D) 250 meters

Q80.A train crosses a bridge 500 meters long in 40 seconds. If the train is moving at a speed of 72 km/h, what is the length of the train?
A) 200 meters
B) 300 meters
C) 400 meters
D) 500 meters

Q81.Two trains, 150 meters and 100 meters long, are moving in opposite directions at speeds of 50 km/h and 70 km/h, respectively. How long will it take them to pass each other?
A) 8.5 seconds
B) 10 seconds

C) 12 seconds
D) 15 seconds

Q82.A train 120 meters long is running at 60 km/h. A man is walking in the opposite direction at 6 km/h. How much time will the train take to pass the man?
A) 6 seconds
B) 8 seconds
C) 10 seconds
D) 12 seconds

Q83.A train is 180 meters long and travels at 90 km/h. A bus, which is 40 meters long, is moving in the same direction at 60 km/h. How much time will it take for the train to pass the bus completely?
A) 12 seconds
B) 18 seconds
C) 20 seconds
D) 22 seconds

Q84. A train moving at a speed of 36 km/h crosses a pole in 12 seconds.What is length of the platform?
A) 130 meters
B) 150 meters
C) 170 meters
D) 180 meters

Q85.A train moving at a speed of 36 km/h crosses a pole in 12 seconds. What is the length of the train?
A) 100 meters
B) 120 meters
C) 130 meters
D) 150 meters

Q86.sleeping" is 108°. What percentage of the day is spent sleeping?
A) 15%
B) 20%
C) 25%
D) 30%

Q87.A pie chart shows the following distribution of students in a class: 50% like cricket, 30% like football, and 20% like basketball. If there are 200 students in total, how many students like football?
A) 40
B) 60
C) 100
D) 120

Q88.In a pie chart, the angle representing "Education" is 90°. What percentage of the total does "Education" represent?
A) 10%
B) 25%
C) 50%
D) 75%

Q89.A pie chart shows the expenditure of a family: 40% on rent, 30% on food, 20% on entertainment, and 10% on transportation. If the total expenditure is ₹50,000, how much is spent on entertainment?
A) ₹5,000
B) ₹8,000
C) ₹10,000
D) ₹12,000

Q90.If the angle of the pie chart representing "Sports" is 72°, what percentage does "Sports" represent in the total?
A) 18%
B) 20%
C) 25%
D) 30%

Q91.In a line chart showing the temperature variation over a week, the temperature on Monday is 25°C, on Tuesday is 28°C, and on Wednesday is 24°C. What is the average temperature for the first three days of the week?
A) 24.5°C
B) 25°C
C) 26°C
D) 27°C

Q92.A line chart shows the sales of a company over four months as follows: January - ₹10,000, February - ₹12,000, March - ₹15,000, April - ₹20,000. What is the percentage increase in sales from January to April?
A) 100%
B) 50%
C) 25%
D) 20%

Q93.From a line chart, the temperature recorded on a day at 9 AM was 30°C, and by 3 PM, it increased to 35°C. What was the rate of change of temperature per hour?
A) 0.5°C
B) 0.75°C
C) 1°C
D) 1.5°C

Q94.In a line chart showing a person's savings over time, the line moves upwards at a steady rate. What does this indicate about the savings?
A) The savings are decreasing.
B) The savings are constant.
C) The savings are increasing at a steady rate.
D) The savings are increasing but at an accelerating rate.

Q95.A line chart shows the sales of a product over a period of 6 months. The line is consistently rising from January to June. What can be inferred from this data?
A) Sales are increasing over time.
B) Sales are constant.

C) Sales are fluctuating.
D) Sales are decreasing over time.

Q96.A pie chart shows the distribution of expenses for a family. The angle for "Rent" is 72°. What percentage of the total expenses does "Rent" represent?

A) 18%
B) 20%
C) 25%
D) 30%

Q97.In a pie chart, the angle for "Education" is 90°. What percentage of the total does "Education" represent?
A) 10%
B) 25%
C) 50%
D) 75%

Q98.A pie chart shows the distribution of students in a class: 40% like cricket, 30% like football, and 30% like basketball. If there are 500 students, how many students like basketball?
A) 100
B) 150
C) 200
D) 250

Q99.A pie chart shows the expenditure of a family: 40% on rent, 30% on food, 20% on entertainment, and 10% on transportation. If the total monthly expenditure is ₹60,000, how much is spent on food?
A) ₹10,000
B) ₹12,000
C) ₹15,000
D) ₹18,000

Q100.In a pie chart, the angle for "Sports" is 54°. What percentage does "Sports" represent?
A) 12%
B) 15%
C) 18%
D) 20%

Answers									
1.B	2.B	3.C	4.B	5.B	6.A	7.A	8.B	9.B	10.A
11.B	12.A	13.D	14.B	15.B	16.B	17.B	18.C	19.B	20.A
21.C	22.B	23.C	24.B	25.B	26.B	27.A	28.B	29.A	30.B
31.A	32.B	33.B	34.C	35.A	36.A	37.B	38.A	39.A	40.B
41.B	42.A	43.B	44.B	45.A	46.A	47.B	48.B	49.A	50.B
51.A	52.B	53.A	54.D	55.A	56.A	57.A	58.B	59.B	60.A
61.A	62.B	63.A	64.A	65.B	66.C	67.B	68.C	69.A	70.B
71.D	72.A	73.B	74.C	75.A	76.B	77.C	78.C	79.B	80.B
81.A	82.B	83.B	84.C	85.B	86.B	87.B	88.C	89.B	90.B
91.B	92.B	93.A	94.C	95.A	96.B	97.B	98.B	99.B	100.C

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