DEPARTMENT OF MATHEMATICS AND STATISTICS

DDU GORAKHPUR UNIVERSITY

GORAKHPUR-273009 (U.P.)

INDIA

Syllabus

based on

National Education Policy- 2020

under

Choice Based Credit System (CBCS)

for

Ph. D. COURSE WORK

in

MATHEMATICS

(Effective from Session 2022-23)

Course Work for Ph. D. Mathematics Students Based on CBCS

The course Work for Ph. D. Mathematics will be spread in only one semester (six months).

Objectives

01. Impart teaching so that the students could develop critical thinking ability about the fundamental aspects of mathematics.

02. Imparting knowledge in research work in various emerging fields of mathematics and its applications.

03. Train the students with mathematical knowledge and computation techniques for carrying out scientific investigations independently.

Subject Prerequisites

To study this subject a student must had the subject(s) Mathematics in class M.A. /M.Sc.

Programme Outcomes (POs)

P01. Knowledge in the topics required for undertaking specialized research in various fields of Mathematical Sciences.

PO2. Identification of unsolved relevant problem in a specific field.

PO3. Articulating ideas and strategies for addressing a research problem.

PO4. Undertaking original research on a particular topic.

PO5. Effectively communicating research, through journal publications and conference presentations to the mathematics community.

PO6. Disseminating research to a broader audience.

PO7. Understand the role of pure and applied mathematics in various fields of Mathematical Sciences.

Programme Specific Outcomes (PSOs)

PSO1. To develop deep understanding of the fundamental axioms in mathematics and capability of developing ideas based on them.

PSO2. To provide advance knowledge of topics in pure mathematics particularly in Analysis, Differential Geometry and Differentiable Manifolds empowering the students to proceed with the area at higher level.

PSO3. To develop the understanding of applied mathematics and motivating the students to use Mathematical modeling techniques as a tool in the study of other scientific domains.

PSO4. To encourage students for research studies in the fields of Differential Geometry, Differential Geometry of Manifolds, Differentiable Manifolds, Structures of Differentiable Manifolds, Tensor Analysis, General Relativity, Cosmology and Gravitation, Mathematical Modeling, Bio Mathematics, Mathematical Biology, Mathematical Epidemiology, Analysis, Real Analysis, Complex Analysis, Functional Analysis, Topology, Wavelet Analysis, Fixed Point Theory, Summability Theory, Number Theory and Cryptography, Coding Theory, Riemannian Geometry, Finsler Geometry, Differential Equations, Integral Transform, Fuzzy Set Theory, Fluid Dynamics, Hydro Dynamics, Algebra, Group Theory, Congruences Theory, Ring Theory, Field Theory, Discrete Mathematics, Optimization Theory, Operations Research, Numerical Analysis, Mathematical Computing, Hydro Statics and related fields.

PSO5. To provide students a wide variety of employment options as they can adopt research as a career or take up teaching jobs or can go for any other profession.

PSO6. To inculcate problem solving skills, thinking and creativity through presentations.

PSO7. To help students in their preparation.

PSO8. To enable the students being life-long learners who are able to independently expand their Mathematical expertise when needed.

Department of Mathematics and Statistics

DDU Gorakhpur University, Gorakhpur

Course Work for Ph. D. (Mathematics) Program

Every student admitted for the Ph. D. program in Mathematics will be required to pass a one semester (six months) course work of minimum 16 credits. The division of this 16 credits course work is in three groups. Group-A (04 credits) courses are compulsory for all Ph. D. students of Mathematics. Group-B (06-credits) courses are discipline-specific courses. Group-C (06 credits) courses are research theme- specific courses. The course work of mathematics is as follows:

Course Nature	Course Code	Course Title	Credit
Group-A/Compulsory Paper			
Compulsory Course	MAT 601	Research Methodology	4+0
			04 Credits
Group-B/Elective Paper (Any one of the following/Opt any one)			
Discipline-Specific Courses	MAT 602	Algebra and Analysis	6+0
	MAT 603	Differential Equations	6+0
	MAT 604	Fluid Dynamics	6+0
	MAT 605	Theory of Optimization and Mathematical Computing	6+0
			06 Credits
Group-C/Elective Paper (Any one of the following/Opt any one)			
Research Theme-Specific Courses	MAT 606	Differential Geometry and General Relativity	6+0
	MAT 607	Mathematical Biology	6+0
	MAT 608	Summability Theory and Wavelet Analysis	6+0
	MAT 609	Fixed Point and Fuzzy Set Theory	6+0
			06 Credits
Thesis			Non-Credit
Total			**16 Credits**

Note: The division of theory and internal marks of each paper will be decided by University (as per common ordinance for examination and assessment).

COURSE CONTENTS

Compulsory Course

Group-A/Compulsory Paper

Course Code	Course Title	Total Credit
MAT-601	**Research Methodology**	**4+0**

Course Objectives: This course is designed to enable students to

1. identify and discuss the issues and concepts salient to the research process.
2. identify and discuss the fundamental knowledge of basics of philosophy of science and ethics, research integrity, publication ethics.
3. identify and discuss the plagiarism tools for a valid and ethical research report.
4. the knowledge internet and its uses in research work.
5. familiar with the MS word, MS-Excel, Power Point and LaTex.

UNIT I

Origin of Research, objectives of research, motivation and necessity of research, Steps in Research, types of research, research approaches, significance and relevance of research, conditions for good research and criteria of good research. What is Research Problem?, selection of research problem, choosing the research area, identification of research problem and solving research problems, role of a research supervisor, Major Research Mile Stones in Ancient Period, Historical Glimpses, Some Contribution of ancient Indian scholars. **(01-Credits)**

UNIT II

Literature review, review of published article and books in the field of research work undertaken, writing a synopsis, writing a research proposal, writing a research paper/article, chapter writing, writing a dissertation, writing a Ph.D. thesis, Review Articles, Proof reading, Keywords and Phrases, bibliography, referencing, Mathematical subject Classifications(MSC) and indexing, short communication, fast track communication of a research paper, Plagiarism prevention, Poster/Oral presentation of research papers, Plenary talks, Invited talks of a conference/ workshop.

(01-Credits)

UNIT III

Ethical issues, ethics with respect to science and research, intellectual honesty and research integrity, copy right, intellectural property right and patent law, plagiarism, citation, indexing of journal, impact factor of journal, h-index, g-index, i10 index, google scholar and acknowledgement. **(01-Credits)**

UNIT IV

Introduction of Internet, Email, MS-Word, MS-Excel, Power Point, LaTex, Beamer in research work. Introduction of at least one Mathematical software out of the following: Mathematica / MATLAB / Maple / Scilab / Sage Math / R programming / Python. Introduction of Math SciNet, ISTOR and other online journals. **(01-Credits)**

Books Recommended:

1. Garg, B.L., Karadia, R., Agarwal, F. and Agarwal, U.K., 2002. An introduction to Research Methodology, RBSA Publishers.
2. Kothari, C.R., 1990. Research Methodology: Methods and Techniques. New Age International, 418p.
3. Day, R.A., 1992. How to write and publish a Scientific paper, Cambridge University Press.
4. Fink, A.,2009. Conducting Research Literature Reviews: From the internet to paper. Sage Publications.
5. Satarkar, S.V.,2000. Intellectual property rights and copyright. EssEss Publications.
6. Saxena, V.P.,2013. Lecture Notes on Research Methodology. Indra Publishing House.
7. Beall, J. (2012) – Predatory publishers are corrupting open access. Nature, 489(7415).
8. P. Chaddah, (2018) Ethics in Competitive Research: Do not get scooped; do not get plagiarized, ISBN:978-9387480865
9. P.K. Sinha & Priti Sinha, Computer Fundamentals, BPB Publications.
10. Dilip Datta, LaTeX in 24 Hours: A Practical Guide for Scientific Writing, Springer (2017).
11. Introduction to Information Technology, ITL Education Solutions, Pearson Education.
12. Introduction to Computer Science, ITL Education Solutions, Pearson Education.
13. George Grätzer, More Math IntoLaTeX, 4th Edition, Springer (2016).
14. Amos R. Omondi, Mark Ng'ang'a, and Ryan Marvin, Python Fundamentals: A Practical Guide for Learning Python, Complete with Real-world Projects for You to Explore, PAKCT Publishing (2019).
15. Trochim, W.M.K, Research Methods: The concise knowledge base, Atomic Dog Publication (2005).
16. Sinha, S.C. and Dhiman, A.K., Research Methodology, Ess Publication (2002).

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. explain key research concepts and issues.

CO 2. have good understanding of publication ethics and scientific conduct.

CO 3. have awareness about indexing and citation databases, open access publications and various research metrics like citations, h-index, Impact Factor etc.

Discipline - Specific Courses

Group-B/Elective Paper

Course Code	Course Title	Total Credit
MAT-602	**Algebra and Analysis**	**6+0**

Course Objectives: The paper of Analysis is introduced to Ph.D. Course work for the study of basic concepts of pure mathematics. The main objective of this paper is to prepare the students for research in analysis, differential geometry, differentiable manifolds, coding theory, algebra and congruences theory.

UNIT I

Group and Congruences Theory: Homomorphism, isomorphism of groups, Kernel of homomorphism, Quotient groups, Fundamental theorem of homomorphism, first, second and third theorem of isomorphism of groups, Basic properties of congruences, Fermat's and Wilson's theorem, The function τ and σ, Euler's ϕ – function, Euler's theorem.

Cryptography: Introduction of cryptography, some simple cryptosystems, additive cipher, shift cipher, caeser cipher, affine cipher, auto key cipher, play fair cipher, hill cipher or enciphering matrices, vigenere cipher, vernam cipher, rail fence cipher, simple columnar cipher, simple columnar with multiple rounds cipher. **(1.5-Credits)**

UNIT II

Real Analysis: Sequences of real number and its convergence, Limit, Continuity, Uniform continuity and Differentiability of function of single variable, Monotonic functions and its properties, Riemann integral, Riemann-Stieltjes integrals, Improper integrals, their classification and convergence, Lebesgue measure and Lebesgue integral.

Complex Analysis: Analytic functions, singularities, complex integration, Riemann surfaces and properties, Power series, Analytic Continuation, Uniqueness of analytic continuation, Maximum Modulus Principle, Schwarz's lemma, Infinite product. **(1.5-Credits)**

UNIT III

Topology: Closed sets. Interior, exterior and boundary points. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology. Continuous functions and homeomorphism, Separable spaces. Seperation axioms (T_0,T_1,T_2,T_3,T_4); Compact sets, Continuous functions and compactness, Sequential compactness, connected spaces, connected sets in R, Continuity of a function and connectedness, Path Connected space, Covering Space.

(1.5-Credits)

UNIT IV

Functional Analysis: Normed linear spaces, Banach spaces, Subspaces and Quotient spaces of normed linear space. Inner product spaces, Hilbert spaces, Parallelogram equality, polarization identity, Separable Hilbert spaces, Grahm-Schmidt orthogonalization process. Inner product structure of dual spaces. Hilbert adjoint operators, Shift operators, Positive operator, normal operators, unitary operators, operator theory. **(1.5-Credits)**

Books Recommended:

1. Aliprantis C.D.,Principle of Real Analysis (third Edition); Academic Press, 1998.
2. Walter Rudin, 'Principles of Mathematical Analysis', Third Edition, Tata McGraw – Hill International book company.
3. Conway J.B. ,Functions of one Complex Variables, Springer/ Narosa, New Delhi.
4. Mark J. Ablowitz and A.S. Fokas: Complex Variables: Introduction and Applications, Cambridge University Press, South Asian Edition, 1998.
5. George F. Simmons : Introduction to Topology and Modern Analysis, McGraw-Hill Book Company.
6. K.D. Joshi: Introduction to General Topology, Wiley Eastern Ltd.
7. P.K. Jain, O.P. Ahuja and K. Ahmad: Functional Analysis, New Age International (P) Ltd. and Wiley Eastern Ltd., New Delhi, 1997.
8. B. Choudhary and S. Nanda: Functional Analysis with Applications, Wiley Eastern Ltd., 1989.
9. Delfs, H., Knebl, H., Introduction to Cryptography, Springer.
10. Niven and Zuckerman: An Introduction to the theory of numbers, Wiley Eastern Ltd.
11. I.N. Herstein : Topics in Algebra, Wiley Eastern Ltd., New Delhi.
12. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul : Basic Abstract Algebra (Second Edition), Cambridge University Press, Indian Edition.

Course Outcomes: After completing the course, the student shall be able to

CO 1. understand the basics of this course.

CO 2. understand the use of this course in different field of mathematical analysis, differential geometry, differentiable manifolds, coding theory and algebra.

CO 3. think and develop new ideas in this course.

Course Code	Course Title	Total Credit
MAT-603	**Differential Equations**	**6+0**

Course objectives: The objective of this course is to make students aware about differential equations of second order, series solution and the emergence of some special functions and their properties. The additional objective of this course is to introduce classification of partial differential equations. This course also relates Fourier and Laplace transforms.

Unit I

Solutions of second order differential equations and emergence of special functions, Bessel's and Legendre's functions, generating function, recurrence relations and orthogonal properties of Bessel's and Legendre's functions. **(1.5-Credits)**

Unit II

Hermite's functions and Chebyshev's functions, generating function, recurrence relations and orthogonal properties of Hermite's and Chebyshev's polynomials, Sturm-Liouville problem, orthogonality of eigen functions of Sturm-Liouville problem. **(1.5-Credits)**

Unit III

Classification of second order partial differential equations, canonical forms for hyperbolic, parabolic and elliptic partial differential equations, method of separation of variables for solving boundary value problems, solutions of Laplace, diffusion and wave equation. **(1.5-Credits)**

Unit IV

Integral transforms, Fourier integral theorem, Fourier sine and cosine integrals, complex form of Fourier integral, Fourier transforms, properties of Fourier transforms, convolution theorem for Fourier transforms, relation between Fourier and Laplace transforms, Fourier transforms of the derivative of functions, applications of Fourier transforms to boundary value problems.

(1.5-Credits)

Books Recommended:

1. V. S. Verma, Series Solution and Special Functions, Neel Kamal Prakashan, Gorakhpur, 2017.
2. V.S. Verma, A Text Book of Partial Differential Equations, Neelkamal Prakashan, Gorakhpur, 2019.
3. B. S Grewal: Higher Engineering Mathematics, Khanna Publication, 2015.
4. P. Prasad and R. Ravindran, Partial Differential Equations, Wiley Eastern, 1985.
5. W.E. Williams, Partial Differential Equations, Oxford Univ. Press, 1980.
6. R.R. Garabedian, Partial Differential Equations, Wiley, 1984.
7. J.N. Sharma, RK Gupta, Special functions, Krishna Prakashan Media (P) Ltd, 2020.
8. V. Lakshmikantham and V. Raghavendra, A text Book of Ordinary Differential Equations, Tata McGraw Hill,1997.

Course Outcomes: After the completion of the course, the students shall be able to

CO 1. learn the series solution of differential equations of second order.

CO 2. use various special functions while solving mathematical problems.

CO3. apply Fourier and Laplace transforms in solving differential equations.

Course Code	Course Title	Total Credit
MAT-604	**Fluid Dynamics**	**6+0**

Course Objectives: The paper of Fluid Dynamics is introduced to Ph.D. Course work for the study of Fluid motion, Lagrangian and Eulerian methods, Euler's and Lagrange's Equation of continuity and equation of motion, Newton's law of viscosity Navier-Stokes equations of motion, Steady viscous flow between parallel planes. The main objective of Fluid Dynamics is that to prepare the students for further research in applied mathematics, cosmology, mathematical modeling and bio mathematics.

Unit I

Fluid dynamics, Basic terminology in fluid motion, Methods of describing fluid motion, Lagrangian and Eulerian methods, Relation between Lagrangian and Eulerian methods, Streamlines, Path lines, Streak lines, Velocity potential, Vorticity vector, Vortex lines, Boundary surface, Equation of continuity by Euler's and Lagrange's methods, Equivalence between Eulerian and Lagrangian forms of equations of continuity, Equation of continuity in other coordinate systems, Symmetrical forms of equation of continuity. **(1.5-Credits)**

Unit II

Euler's and Lagrange's equation of motion, Lamb's hydrodynamical equations, Conservative field of force, Euler's equations of motion in cylindrical and Spherical polar coordinates, Equations of motion under impulsive force, Energy equation, Pressure equation, Bernaulli's equation and its applications, Euler's momentum theorem, D'Alermbert's paradox. **(1.5-Credits)**

Unit III

Newton's law of viscosity, Kinds of fluids, Nature of stress. Stress components in a real fluid, Symmetry of stress tensor, Transformation of stress components, Stress invariants, Relations between Cartesian components of stress, Rate of strain quadric, Principal stresses, Stoke's law of viscosity, Relations between stress and rate of strain. **(1.5-Credits)**

Unit IV

General motion of a fluid elements, Navier-Stokes equations of motion, Steady viscous flow between parallel planes. Steady flow through a tubes of uniform circular cross-sections. Steady flow between concentric rotating cylinders, Unsteady flow of viscous incompressible fluid between two parallel plates, Pulsatile flow between parallel surfaces, Reynold's number and its physical significance. **(1.5-Credits)**

Recommended Books:

1. J.K. Goyal and K.P. Gupta: Fluid Dynamics, Pragati Prakashan, Meerut, 2017.
2. N. Curle and H. J. Davis: Modern Fluid Dynamics, D. Van Nostrand Company Ltd. London, 1968.
3. G.K. Batchelor: An Introduction to Fluid Dynamics, Cambridge University Press, Cambridge, 2000.
4. F. Chorlton: A Text Book of Fluid Dynamics, CBS Publishers and Distributors, New Delhi, 2002.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. effectively write mathematical solutions in a clear and concise manner.

CO 2. demonstrate an intuitive and computational understanding of Fluid motion, Lagrangian and Eulerian methods, Euler's and Lagrange's Equation of continuity, Newton's law of viscosity Navier-Stokes equations of motion, Steady viscous flow between parallel planes.

CO 3. research in applied mathematics, cosmology, mathematical modeling and bio mathematics.

Course Code	Course Title	Total Credit
MAT-605	**Theory of Optimization and Mathematical Computing**	**6+0**

Course Objectives: The paper of theory of optimization and mathematical computing is introduced to Ph.D. Course work for the study of basic concepts of optimization and computing theory. The main objective of this paper is to prepare the students for research in optimization and mathematical computing theory.

UNIT I

Inventory Control: Introduction, Classification of Inventory, Economic parameter associated with inventory problems, Deterministic and Probabilistic models with without lead time.

Sequencing and Replacement Problems: Assumptions for seqnencing problem. Processing n jobs on two machines,n jobs on three machines, 2 jobs on m machines, Problem of Replacement, Individuals and Group replacement policies. **(1.5-Credits)**

UNIT II

Network Analysis: Basic concepts and definition. Network drawing and analysis Critical path method. Labelling method. Methods based on time estimates to find critical path. Concept of slack and float. Resource levelling and time-cost trade-off analysis. Time-cost optimization procedure, Project crashing. PERT, Diffferences in PERT and CPM. Shortest path model minimum spanning tree problems. Kruskal's and Prim's Algorithm**.** **(1.5-Credits)**

UNIT III

Numerical Methods: Curve fitting, Method of Least squares, fitting of straight-line, second-degree polynomial, power curve, exponential curve etc., Interpolation with equal and unequal intervals, Newton's forward and backward interpolation formulae, Divided difference interpolation formula, Lagrange's interpolation formula, Hermite's interpolation formula, Solution of system of linear equations by Gauss Jacobi method, Guass Siedel method, Relaxation method and Crout's method and Crout's method for finding the inverse of matrix, Cholesky method, Numerical Double Integration by Trapezoidal and Simpson rules**.** **(1.5-Credits)**

UNIT IV

Mathematical Computing Software R/Python: Introduction computer programming, Data Types, Variables, basic operators, Boolean values, loops, logical operations, Functions, Data Processing, package, the object orientated approach: classes methods, objects, the standard objective features, Exception Handling, working with files, Basic mathematical calculation such as differentiation, integration, etc. **(1.5-Credits)**

Books Recommended:

1. S.D. Sharma: Operations Research, Kedar Nath Ram Nath & Company.
2. S.S. Rao: Optimization Theory and Applications, Wiley Eastern Ltd., New Delhi.
3. J.K. Sharma: Operations Research – Theory and Applications, Macmillan India Ltd.
4. Winston Wayne L., Operations Research: Applications and Algorithms, Cengage Learning, 4th Edition.
5. H.A. Taha: Operations Research – An Introduction, Macmillan Publishing Co., Inc., New York.
6. M. K. Jain, S.R.K. Iyengar & R.K. Jain, Numerical Methods for Engineering and scientific computation.
7. S. S. Sastry, Introductory methods of Numerical Analysis.
8. Eric Matthes, Python Crash Course, William Polock 2016.
9. Amos R. Omondi, Mark Ng'ang'a, and Ryan Marvin, Python Fundamentals: A Practical Guide for Learning Python, Complete with Real-world Projects for You to Explore, PAKCT Publishing (2019).
10. Garrett Grolemund Hands–On Programming with R: Write Your Own Functions and Simulations, O'Reilly Publication (2014).

Course Outcomes: After completing the course, the student shall be able to

CO 1. understand the basics of this course.

CO 2. understand the use of this course in different field of mathematics for optimization and computation.

CO 3. understand some advanced computing tools and techniques for graphical representation of data, data analysis in their research work.

Research Theme- Specific Courses

Group-C/Elective Paper

Course Code	Course Title	Total Credit
MAT-606	**Differential Geometry and General Relativity**	**6+0**

Course Objectives: The paper of Differential Geometry and General Relativity is introduced to Ph.D. Course work for the study of basic concepts of differential geometry and general relativity. The main objective of differential geometry and general relativity is to prepare the students for research in tensor analysis, differential geometry, differentiable manifolds, structures of differentiable manifolds, riemannian geometry, finsler geometry, analysis, general relativity and cosmology.

UNIT I

Tensor Analysis: Transformation of coordinates, Contravariant and Covariant vectors and tensors, Mixed tensors, Algebra of tensors, Riemannian Metric, Christoffel symbols, Covariant differentiation of vector and tensor, Riemannian curvature tensor, Flat and Einstein space.

Differentiable Manifolds: Differentiable manifold, Lie bracket, Connections, Torsion, Curvature, Lie derivative, Riemannian Manifold, Riemannian connection, Riemannian curvature tensor, Ricci tensor, Projective, conformal, concircular and conharmonic curvature tensors, Einstein manifold, Flat Manifold, Exterior derivative, Recurrent and symmetric manifold, Killing vector field.

(1.5-Credits)

UNIT II

Complex and Contact Manifolds: Almost Complex Manifolds, Nijenhuis tensor, Almost Hermite Manifolds, Kaehler Manifolds, Almost Contact Manifolds, Almost Grayan Manifolds, K-contact Riemannian manifold, Sasakian Manifolds, Properties of Projective, Conformal, Concircular and Con- harmonic curvature tensor in Sasakian manifold. **(1.5-Credits)**

UNIT III

General Relativity: Space time, Curved Space time, Introduction of General Relativity, Equivalence Principle, Principle of General Covariance, Mach's Principle, Einstein Field equations, Schwarzschild exterior solution, Energy momentum Tensor, Birkhoff's Theorem, General Relativity Crucial Tests. **(1.5-Credits)**

UNIT IV

Cosmology: Static and Non-Static Models, Properties of Einstein and de-Sitter Universe, Friedmann-Robertson-Walker Cosmological models, Particle and Event Horizon, Life cycle of stars, Gravitational Collapse, Spherically symmetric gravitational collapse of star: dust, perfect fluid, Non-Spherical Gravitational Collapse, Black Holes, Hubble hypothesis, Singularity in Cosmological models**.** **(1.5-Credits)**

Books Recommended:

1. R.S. Mishra: Structure on differentiable manifold and their application, Chandrama Prakashan, Allahabad, 1984.
2. K. Yano and M. Kon: Structures of Manifolds, World Scientific Publishing Co. Pvt. Ltd., 1984.
3. R. S. Mishra: A Course in Tensors with Applications to Riemanian Geometry, Pothishala, Allahabad, 1965.
4. U.C.De and A.A.Shaaikh: Complex and Contact Manifolds, Narosa Publishing House, New Delhi 2009.
5. K.S.Amur, D.J.Shetty and C.S.Bagewadi: An Introduction to Differential Geometry, Narosa Publishing House, New Delhi 2010.
6. S. Shahshahani: An Introductory Course on Differentiable Manifolds, Dover Publication Inc. New York, 2016.
7. B. B. Sinha, An Introduction to Modern Differential Geometry, KalyaniPrakashan, New Delhi, 1982.
8. David C. Kay, Tensor Analysis, Schaum's Outline Series, McGraw Hill 1988.
9. N.J. Hicks, Notes on Differential Geometry, D. Van Nostrand Inc., 1965.
10. U.C.De and A.A.Shaaikh, Differential Geometry of Manifolds, Narosa Publishing House, New Delhi 2007.
11. K. D. Krori : Fundamentals of Special and General Relativity; PHI Publication, 2010.
12. S. R. Roy and Raj Bali : Theory of Relativity; Jaipur Publishing House, 2008.
13. Steven Weinberg : Gravitation and Cosmology : Principles and applications of General Relativity; Wiley Publ.,2005.
14. J. V. Narlikar : An Introductions to Relativity; Cambridge University Press, 2010.
15. I.B. Khriplovich : General Relativity; Springer Science + business media, 2005

Course Outcomes: After completing the course, the student shall be able to

CO 1. understand the basics of this course.

CO 2. understand the use of this course in different field of tensor analysis, differential geometry, analysis, differentiable manifolds, general relativity, gravitation and riemannian geometry.

CO 3. think and develop new ideas in differential geometry, analysis, structures of differentiable manifolds, riemannian geometry, finsler geometry, general relativity and cosmology.

Course Code	Course Title	Total Credit
MAT-607	**Mathematical Biology**	**6+0**

Course Objectives: The objective of this course is to enable students to know about mathematical modelling, its need and different tools to formulate mathematical models representing real-world phenomenon and to build mathematical models for some diseases.

Unit I

Mathematical models, need, techniques and classification of mathematical models, limitations of mathematical modelling, linear and non-linear growth and decay models. **(1.5-Credits)**

Unit II

Autonomous systems, critical points, types of critical points, stability of critical points, critical points and stability for linear systems, stability of Lyapunov's direct method, simple critical points of non-linear systems, non-linear mechanics: conservative systems. **(1.5-Credits)**

Unit III

Introduction to difference equations, solution of difference equations, simultaneous difference equations with constant coefficients, solution of linear difference equations by Laplace transform and Z-transform, solution of nonlinear difference equations reducible to linear difference equations, applications of difference equations in mathematical modelling, stability theory for difference equations. **(1.5-Credits)**

Unit IV

BLL model, Leslie matrix, determination of eigen values and eigen vectors of Leslie matrix. introduction of mathematical epidemiology, basic terminologies, SI model, SIS model with constant coefficients, SIS model with constant number of carriers, Kermack-McKendrick SIR model. **(1.5-Credits)**

Books Recommended:

1. J.N. Kapur: Mathematical Models in Biology and Medicine, Affiliated East-West Press Pvt. Ltd., New Delhi, 1985.
2. G.F. Simmons: Differential Equations with Applications and Historical Notes, McGraw Hill Education India,1991.
3. J. Mazumdar: An Introduction to Mathematical Physiology and Biology, Cambridge University Press, 1999.
4. Nicholas F. Britton: Essential Mathematical Biology, Springer, 2003.

Course Outcomes: After the completion of the course, the student shall be able to

CO 1. assess and articulate what type of modelling techniques are appropriate for a given system.

CO 2. construct mathematical models in various real-world phenomenon and analyze them.

CO 3. study the spread and control of some diseases through mathematical modeling.

Course Code	Course Title	Total Credit
MAT-608	**Summability Theory and Wavelet Analysis**	**6+0**

Course Objectives: The paper of Summability Theory and Wavelet Analysis is introduced to Ph.D. Course work for the study of basic concepts of summability and wavelet analysis. The main objective of summability theory and wavelet analysis is to prepare the students for research in analysis.

Unit-I

The Fourier series of a periodic function, Convergence problem, Dirichlet's integral, n^{th} partial sum of Fourier series and its conjugate series, Riemann-Lebesgue theorem, convergence tests, Dini's test, Jordan's test, dela Vallee-Poussin's test, relation between the tests of Dini' Jordan and de la Vallee-Poussin tests. **(1.5-Credits)**

Unit-II

Arithmetic means, Summation of a series by Arithmetic Mean, Special methods of summation: Nörlund means, Regularity and Consistency of Nörlund means, Inclusion, Equivalence. Euler's means, Abelian means, Riesz's typical means., Hölder's means, simples theorems concerning Hölder summability, Cesàro means, means of non-integral orders, simple theorems concerning Cesàro summability , Cesàro and Abel summability, Cesàro means as Nörlund means, Tauberian theorems for Cesàro summability. **(1.5-Credits)**

Unit-III

Fourier and inverse Fourier transforms, Convolution and delta function, Fourier transform of Square integrable functions. Fourier series, Poisson's Summation formula. **(1.5-Credits)**

Unit-IV

Wavelet Transforms and Time Frequency Analysis: The Gabor Transform, Short-time Fourier transforms and the uncertainity principle. The integral wavelet transforms Dyadic wavelets and inversions, Frames. Wavelet Series. Scaling Functions and Wavelets: Multi resolution analysis, scaling functions with finite two scale relations. Direct sum decomposition of L^2 (R), Orthogonal Wavelets, Example of orthogonal Wavelets. **(1.5-Credits)**

Books Recommended:

1. Trigomometric Series Vol.1&2, A. Zygmund: Cambridge University Press.
2. Divergent Series, Hardy G. H, Oxford Clarendon Press.
3. Fourier Analysis, Javier Duoandikoetxea, American Mathematical Society.
4. C.K.Chui, A first course in wavelets, Academic press NY 1996.
5. Daubechies, Ten lectures in wavelets, Society for Industrial and Applied Maths, 1992.

Course Outcomes: After completing the course, the student shall be able to

CO 1. understand the basics of this course.

CO 2. understand the use of this course in analysis.

CO 3. think and develop new ideas in this course.

Course Code	Course Title	Total Credit
MAT-609	**Fixed Point and Fuzzy Set Theory**	**6+0**

Course Objectives: The paper of Fixed Point Theory in Ph.D. course work is intended as a brief introduction to the subject with a focus on fixed point theorems for single valued and multivalued mappings and its applications to differential and integral equations. Fuzzy set theory Provides an understanding of the basic mathematical elements of the theory of fuzzy sets, soft sets and information system. This course provides an emphasis on the differences and similarities between fuzzy set and classical set theories. We can got solution of the problems of decision making and information system in fuzzy set.

Unit-I

Background of Metrical fixed point theory, Fixed Points, Uniformly convex, strictly convex and reflexive Banach spaces, Lipschitzian and contraction mapping, Banach's contraction principle, Application to Volterra and Fredholm integral equations, Caristi's fixed point theorem. Nonexpansive, asymptotically nonexpansive and quasinonexpansive mappings and Fixed Points, Fixed point theorems for nonexpansive mappings, Nonexpansive operators in Banach spaces satisfying Opial's conditions, The demiclosedness principle**.** **(1.5-Credits)**

Unit-II

Brouwer's fixed point theorem, Schauder's fixed point theorem, Measure of NonCompactness, Condensing map, Fixed points for condensing maps, Strict convexity, Uniform convexity, The modulus of convexity and normal structure, Smoothness, retraction map, Sadovskii's fixed point theorem, Introduction of Set-valued mappings, Set-valued contraction map, Housdroff metric, Nadler's fixed point theorem. **(1.5-Credits)**

Unit-III

Overview of Crisp set, Concept of Fuzzy Set, Different types of fuzzy set, convex fuzzy set, cuts, Respresentation of fuzzy sets, Extension principle for fuzzy sets, Standard Operations on Fuzzy Set: Fuzzy Complement, Fuzzy Union & Fuzzy Intersection, T-norms and T-conorms, Combination of operations, Fuzzy number, Operations on Intervals, Operations on fuzzy numbers, Lattice of fuzzy numbers, Fuzzy equation, Fuzzy relation. **(1.5-Credits)**

Unit-IV

Introduction of soft set, Fuzzy set: a special case of soft set, Reduct soft set operations on soft sets, Soft set relation and function, Soft expert set, Multisoft set Introduction to fuzzy soft set, Matrix representation of fuzzy soft set, Fuzzy soft set relation and function. Information system, Soft set: as an information system, Representation of information system as soft set, Multisoft sets in information system. Individual decision making, Multi-person decision making, Application of generalized fuzzy soft sets in decision making. **(1.5-Credit)**

Books Recommended:

1. V. Berinde, Iterative Approximation of Fixed Points, Lecture Notes in Mathematics, No. 1912, Springer, 2007.
2. M. A. Khamsi and W. A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, John Wiley & Sons, New York, 2001.
3. Sankatha P. Singh, B. Watson and P. Srivastava, Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1997.
4. V. I. Istratescu, Fixed Point Theory: An Introduction, D. Reidel Publishing Co., 1981.
5. K. Goebel and W. A. Kirk, Topic in Metric Fixed Point Theory, Cambridge University Press, 1990.
6. George J. Klir, Bo Yuan, Fuzzy Sets and Fuzzy Logic Theory and Applications.
7. H.J. Zimmermann, Fuzzy Set Theory and Its Applications.

Course Outcomes: At the end of the course, student will be able to

CO 1. understand the basics of this course.

CO 2. cover a wide area of research in fixed point theory and its applications in different fields of science.

CO 3. distinguish between the crisp set and fuzzy set concepts through the learned differences between the crisp set characteristic function and the fuzzy set membership function. By theory of fuzzy and soft sets in decision making and information system we can solve the problems of local industries, game theory and computer programming.