# Assignment Problem

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The name ***assignment problem*** originates from the classical optimization problems where the objective is to assign a number of origins (jobs) to the equal number of destinations (machines) at a minimum cost or at a maximum profit. In other words, an assignment problem is an optimization problem in which an assignment schedule (i.e. which job should be assigned to which machine) is sought to minimize the effective cost or maximize the effective profit. Indeed, we are often faced with the problems of assigning jobs to persons, jobs to machines, workers to machines, classes to class-rooms, drivers to trucks, etc., where the assignees possess varying degrees of efficiency of work so as to make the cost involved minimum.

## Nature and Mathematical Formulation of an Assignment Problem

We restrict our discussions to assignment problems in which there are as many jobs as the number of machines so that the jobs can be assigned to machines in one-to-one way at minimum effective cost.

To examine the nature of assignment problem, let there be n jobs and n machines and let $c_{ij}$ be the cost (payment) of assigning the $i^{th}$ job to the $j^{th}$ machine. Then, the problem is to determine which job should be assigned to which machine so that the total effective cost is minimum. The matrix $[c_{ij}]$ is called as the ***cost matrix*** or ***effectivenessmatrix*** of the assignment problem that is $c_{ij}$ is the ***cost*** or ***effectiveness*** of assigning the $i^{th}$ job to the $j^{th}$ machine.

To put the above problem in a mathematical form, we define a set of variables $x_{ij}$ as follows:

$$x_{ij} = \begin{cases} 1, \text{if the} i^{th} \text{ job is assigned to the} j^{th} \text{ machines} \\ 0, \text{otherwise} \end{cases}$$

Then, clearly we have $\sum_{i=1}^{n} x_{ij} = 1 = \sum_{j=1}^{n} x_{ij}$ ; as one job is assigned to one.

machine only.

Mathametically, the assignment problem can be stated as follows :

$$\text{Minimize Z} = \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}\, c_{ij} \;; i = 1,2, \dots, n; j = 1,2, \dots, n.$$

subject to the following constraints :

$$x_{ij} = \begin{cases} 1, \text{if the} i^{th} \text{ job is assigned to the} j^{th} \text{ machines} \\ 0, \text{otherwise} \end{cases}$$

$\sum_{j=1}^{n} x_{ij} = 1; i = 1,2, \dots, n;$ [since only one job is assigned to the $j^{th}$ machines]

$\sum_{i=1}^{n} x_{ij} = 1; j = 1,2, \dots, n; j = 1,2, \dots, n.$ [since only one machine should be assigned the $j^{th}$ job

where $x_{ij}$ denotes that the $i^{th}$ job is (or to be) assigned to the $j^{th}$ machineand Z is the total assignment cost. Obviously, we have $x_{ij} \geq 0$; $i$ = 1, 2, ...,$n$; $j$ = 1, 2, ..., $n$. This problem is explicitely represented by the following $n \times n$ cost matrix :

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| | | $M_1$ | $M_2$ | | $M_J$ | | $M_n$ |
|---|---|---|---|---|---|---|---|
| | $J_1$ | $c_{11}$ | $c_{12}$ | | $c_{ij}$ | | $c_{1n}$ |
| | $J_2$ | $c_{21}$ | $c_{22}$ | | $c_{2j}$ | | $c_{2n}$ |
| Jobs | $J_3$ | | | | | | |
| | $J_4$ | $c_{i1}$ | $c_{i2}$ | | $c_{ij}$ | | $c_{in}$ |
| | $J_5$ | | | | | | |
| | $J_6$ | $c_{n1}$ | $c_{n2}$ | | $c_{nj}$ | | $c_{nn}$ |

Thus, an assignment problem is apparently a general linear programming problem, but many mathematicians refuse to accept this as a linear programming problem as the decision variables are only zero-valued and one-valued.

**Remarks :**(i) Though the method is developed here for job machine assignments, it can very well be applied to other similar problems like worker-machine assignments, class-classroom assignments, driver-truck assignments, etc.

(ii) The method can be extended to even rectangular problems i.e. problems having the number of jobs different from the number of machines with a slight modification.

## Assignment Problem as a Special Type of Transportation Problem

An assignment problem can be looked upon as a special type of an (mxn)-transportation problem in which the jobs stand for origins (sources), the machines for destinations and all the availabilities and requirements are equal to one. In such a case, $m = n$ and all $a_i$ and $b_j$ are unity and each $x_{ij}$ is limited to one of the two values 0 and 1. It can be seen that exactly $n$ of $x_{ij}$ can be non-zero (i.e. unity), one for each origin and one for each destination.

## Method of Solution of an Assignment Problem

Here, we discuss a specific method of solution of an assignment problem. Before explaining the method of solution, we need the following theorems :

### Fundamental Theorems of Assignment Problem

The solution to an assignment problem is fundamentally based on the following theorems :

**Theorem 1. (Reduction Theorem) :** If a constant is added (or subtracted) to (or from) any row or column of the cost matrix of an assignment problem, then an assignment schedule (plan) that minimizes the total cost for the new cost matrix also minimizes the total cost for the original cost matrix.

**Proof :**Let a constant λ be added (or subtracted) to (or from) the $k^{th}$ row of the cost matrix $[c_{ij}]$, where $1 \leq k \leq n$, of the following original assignment problem :

$$\text{Minimize Z} = \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}\, c_{ij}\ ; i = 1,2,\ldots,n; j = 1,2,\ldots,n.$$

subject to the following constraints :

$$x_{ij} = \begin{cases} 1, \text{if the} i^{th} \text{ job is assigned to the} j^{th} \text{ machines} \\ \qquad\qquad 0, \text{otherwise} \end{cases}$$

$$\sum_{j=1}^{n} x_{ij} = 1; i = 1,2,\ldots,n; \qquad \sum_{i=1}^{n} x_{ij} = 1; j = 1,2,\ldots,n;$$

where$x_{ij}$denates that the $i^{th}$ job is (or to be) assigned to the $j^{th}$ machine and Z is the total assignment cost for the original cost matrix $[c_{ij}]$.

If Z′denotes the total assignment cost for the changed cost matrix afteradding (or subtracting) the constantλ, then we have :

$$\mathrm{Z}' = \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}\left(c_{ij} + \lambda\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}\, c_{ij} \pm \sum_{i=1}^{n}\sum_{j=1}^{n} \lambda x_{ij}$$

$$= \mathrm{Z} \pm \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij} = \mathrm{Z} \pm \sum_{j=1}^{n}\left(\lambda \sum_{j=1}^{n} x_{ij}\right)$$

$$= Z \pm \sum_{i=1}^{n} \lambda . 1 = Z \pm \lambda$$

Thus, if Z is minimum for a set of values of $x_{ij}$ of the original assignmentproblem, thenZ′is also minimum for the same set of values $x_{ij}$ of the new or changed or (reduced) assignment problem, as Z′differs from Z only by aconstantλ.

This indicates that if different constants are added to different rows and different columns, then also, the optimal solution is not changed for the same reason as above.

**Theorem 2.**If there exists a set of values $\{x_{ij}*\}$ for an assignment problem with cost matrix $[c_{ij}]$, where $c_{ij} \geq 0$ such that $\sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}\, c_{ij} = 0$, then $\{x_{ij}*\}$ is an optimal solution of the problem.

**Proof :** Since $c_{ij} \geq 0$ and $x_{ij}* \geq 0$ for all $i$ and $j$, therefore, the objective function or the assignment cost Z $= \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}\, c_{ij}$ cannot be negative. The minimum possible value that Z can attain, is zero.

Thus, any $x_{ij}*$ of the set of feasible solutions $\{x_{ij}*\}$ which satisfies the condition $\sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}\, c_{ij} = 0$, will be optimal and so $\{x_{ij}*\}$ is an optimal solution.

## Computational Procedure of Method of Solution of Assignment Problem: Hungarian Assignment Method (or Reduced Matrix Method)

The assignment problem having $n \times n$ cost matrix constitutes $n!$ possible ways of assigning $n$ jobs to $n$ machines . If we enumerate all these $n!$ alternatives (ways) and evaluate the cost of each one (alternative) of them and select the one with the minimum cost, then the problem is said to be solved. But this method is very slow and time consuming even for small value of n and hence it is not suitable at all. However, a much more efficient method of solving such problems is available. This is ***Hungarian Assignment Method*** of solving assignment problems with $n \times n$ cost matrix which is based on the reduction of the cost matrix. Therefore, this method is sometimes also called as ***Reduced Matrix Method***. This method was developed and represented by Hungarian Mathematician ***D. König***.

Various steps of the computational procedure of Hungarian Method for obtaining an optimal assignment to an assignment problem are as follows :

**Step 1.** Subtract the smallest (least) element of each row of the given cost matrix from all elements of that row. Now, examine whether each row contains at least one zero or not. If not, subtract the smallest element of each column (not containing zero) from all the elements of that column. The matrix thus obtained is often called the ***reduced matrix*** or ***reduced table.***

Note that we can start first with columns instead of rows and then go to rows to obtain the reduced matrix.

**Step 2.**Cross out all zeros by drawing the minimum number of horizontal and/or vertical lines.

Note that this number does not exceed the order $n$ of the cost matrix. If this number is equal to n, then the optimal assignment has been reached and we go to step 3.

**Step 3.** Starting with the first row, determine the rows containing only one zero and box (or enrectangle) them. Do the same for the columns. If these row and column operators do not give complete assignment, then draw both horizontal and vertical lines through the boxed (enrectangled) zeros and examine the reduced matrix (or table) once more as above. If even after this, multiple (i.e. more than one) zeros exist in a row or column, then choose any one of these zeros and box (enrectangle) it. The horizontal and vertical lines drawn through this, will then give unique zero in the reduced matrix. Such a case infers an ***alternative solution***. (See illustrative example 3.).

**Step 4.** If the minimum number of horizontal or vertical lines or both crossing out all zeros is less than the order n of the cost matrix, then subtract the smallest uncrossed cost element from each of the uncrossed cost elements, add the same to the cost element at the intersection of the horizontal and vertical lines (generally called as ***crossing cost element***) and leave the other crossed cost elements (crossed by a single line only) as they are.

**Step 5.** With the modified matrix obtained in step 4, repeat or pursue the step 2. and step 3 until the minimum number of horizontal and vertical lines become equal to the order $n$ of the cost matrix.

**Step 6.** Examine the rows successively until a row-wise exactly single zero is found. Mark this zero and put it in a box (☐) to make the assignment. Then, mark a cross (×) over all zeros lying in the column of the marked zero in the box ☐, showing that they cannot be considered for future assignment. Continue in this manner until all the rows have been examined. Repeat the same procedure for columns also.

**Step 7.** Repeat the step 6. successively until one of the following situations arise :

**(i)** if no unmarked zero is left, then the process ends or **(ii)** if there lie more then one of the unmarked zeros in any column or row, then mark one of the unmarked zero arbitrarily and put it in the box (☐) and mark a cross in the cells of remaining zeros in its row and column.

Repeat the process until no unmarked zero is left in the reduced cost matrix.

**Step 8.** Obtain exactly one marked zero in the box (☐) in each row and in each column of the reduced cost matrix.

The assignment corresponding to these marked positions (i.e. marked zeros in the box '☐') will give the optimal assignment and the total optimal assignment cost is given by the sum of cost elements to the marked positions (i.e. marked zeros in the box '☐').

## Rules to Draw the Minimum Number of Horizontal and Vertical Lines

A very convenient rule of drawing the minimum number of horizontal and vertical lines to cross-out all the zeros of the modified (i.e. reduced) matrix is given below in various steps :

**Step 1.** Tick (✓) the rows that do not have any marked zeros in a box '☐'.

**Step 2.** Tick (✓) the columns having marked zeros in a box '☐' or otherwise in ticked rows.

**Step 3.** Tick (✓) the rows having marked zeros in the ticked columns.

**Step 4.** Repeat step 2. and step 3. until the chain of ticking is complete.

**Step 5.** Draw the horizontal and vertical lines through all unticked rows and ticked columns.

The following examples will illustrate the ***Hungarian Assignment Method*** best :

| | | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|---|
| | $J_1$ | 12 | 24 | 15 |
| Jobs | $J_2$ | 23 | 18 | 24 |
| | $J_3$ | 30 | 14 | 28 |

**Solution :**The smallest cost elements in the 1st, 2nd and 3rd rows are 12, 18 and 14 respectively. Subtracting the smallest cost 12 from all elements of the 1st row, the smallest cost 18 from all elements of the 2nd row and the smallest cost 14 from all elements of the 3rd row, we get

| | | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|---|
| | $J_1$ | 0 | 12 | 3 |
| Jobs | $J_2$ | 5 | 0 | 6 |
| | $J_3$ | 16 | 0 | 14 |

Since, 0, 0 and 3 are the smallest cost elements in the 1st, 2nd and 3rd columns respectively, therefore, we subtract 0, 0 and 3 from the respective columns and obtain the following :

| | | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|---|
| | $J_1$ | 0 | 12 | 0 |
| Jobs | $J_2$ | 5 | 0 | 3 |
| | $J_3$ | 16 | 0 | 11 |

Now, the number of horizontal and vertical lines required to cross out all the zeros is 2 which is less than the order 3 of the cost matrix. Clearly, 3 is the smallest cost among all uncrossed cost elements. So, subtracting 3 from the uncrossed cost elements and adding 3 to the crossing cost element 12, we get the following modified matrix :

| | | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|---|
| | $J_1$ | 0 | 15 | 0 |
| Jobs | $J_2$ | 2 | 0 | 0 |
| | $J_3$ | 13 | 0 | 8 |

Clearly, the minimum number of lines required to cross-out all zeros is 3. Hence, the optimal stage of assignment is reached. To find out the specific assignments, we start setting boxes to the single zeros in rows and columns and obtain the final assignment as shown below :

| | | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|---|
| | $J_1$ | $\boxed{0}$ | 15 | $\cancel{0}$ |
| Jobs | $J_2$ | 2 | $\cancel{0}$ | $\boxed{0}$ |
| | $J_3$ | 13 | $\boxed{0}$ | 8 |

Hence, the optimal assignment schedule is :$J_1 \rightarrow M_1$, $J_2 \rightarrow M_3$ and $J_3 \rightarrow M_2$

The minimum assignment cost i.e. the optimal cost of assignment = Rs. (12 + 24 + 14) = Rs. 50.

**Example 2.** Solve the following assignment problem (where the costs are given in terms of Rs.) :

| | I | II | III | IV |
|---|---|---|---|---|
| 1 | 1 | 4 | 6 | 3 |
| 2 | 9 | 7 | 10 | 9 |
| 3 | 4 | 5 | 11 | 7 |
| 4 | 8 | 7 | 8 | 5 |

**Solution :**The smallest cost elements in the 1st, 2nd, 3rd and 4th rows are 1,7,4 and 5 respectively. Subtracting the smallest cost elements 1,7,4 and 5 respectively from all the cost elements of the 1st, 2nd, 3rd and 4th rows, we get

| | I | II | III | IV |
|---|---|---|---|---|
| 1 | 0 | 3 | 5 | 2 |
| 2 | 2 | 0 | 3 | 2 |
| 3 | 0 | 1 | 7 | 3 |
| 4 | 3 | 2 | 3 | 0 |

Since 0, 0, 3 and 0 are the smallest cost elements in the Ist, 2nd, 3rd and 4th columns respectively. Therefore, we subtract the smallest cost elements 0,0,3 and 0 from the respective columns and obtain the following:

| | I | II | III | IV |
|---|---|---|---|---|
| 1 | 0 | 3 | 2 | 2 |
| 2 | 2 | 0 | 0 | 2 |
| 3 | 0 | 1 | 4 | 3 |
| 4 | 3 | 2 | 0 | 0 |

Now, the number of horizontal and vertical lines required to cross out all the zeros is 3 which is less than the order 4 of the cost matrix. Here, 1 is the smallest cost among all uncrossed cost elements. Hence, we subtract 1 from all uncrossed cost elements and add 1 to the crossing cost elements to obtain the following :

| | I | II | III | IV |
|---|---|---|---|---|
| 1 | 0 | 2 | 1 | 1 |
| 2 | 3 | 0 | 0 | 2 |
| 3 | 0 | 0 | 3 | 2 |
| 4 | 4 | 2 | 0 | 0 |

Since, the minimum number of lines required to cross out all zeros is 4. Hence, the optimal stage of assignment is reached. To find out the specific assignments, we start setting boxes to the single zeros in rows and columns and obtain the final assignment as shown below :

Hence, the optimal assignment schedule is : 1 → I, 2 → III, 3 → II and 4 → IV.

The minimum cost of assignment = Rs. (1 + 10 + 5 + 5) = Rs. 21.

**Example 3.** Find the optimal assignment corresponding to the following cost matrix (where the costs are given in terms of Rs.) :

| | I | II | III | IV |
|---|---|---|---|---|
| 1 | $\boxed{0}$ | 2 | 1 | 1 |
| 2 | 3 | $\cancel{0}$ | $\boxed{0}$ | 2 |
| 3 | $\cancel{0}$ | $\boxed{0}$ | 3 | 2 |
| 4 | 4 | 2 | $\cancel{0}$ | $\boxed{0}$ |

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| I | 9 | 8 | 7 | 6 | 4 |
| II | 5 | 7 | 5 | 6 | 8 |
| III | 8 | 7 | 6 | 3 | 5 |
| IV | 8 | 5 | 4 | 9 | 3 |
| V | 6 | 7 | 6 | 8 | 5 |

**Solution :** The smallest cost elements in the 1st, 2nd, 3rd, 4th and 5th rows are 4, 5, 3, 3 and 5 respectively. Subtracting the smallest cost of each row

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| I | 5 | 4 | 3 | 2 | 0 |
| II | 0 | 2 | 0 | 1 | 3 |
| III | 5 | 4 | 3 | 0 | 2 |
| IV | 5 | 2 | 1 | 6 | 0 |
| V | 1 | 2 | 1 | 3 | 0 |

from all the cost elements of the corresponding rows, we get

Again, subtracting the smallest cost of each column from all the cost elements of the corresponding columns, we obtain

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| I | 5 | 2 | 3 | 2 | 0 |
| II | 0 | 0 | 0 | 1 | 3 |
| III | 5 | 2 | 3 | 0 | 2 |
| IV | 5 | 0 | 1 | 6 | 0 |
| V | 1 | 0 | 1 | 3 | 0 |

Now, we see that the minimum number of horizontal and vertical lines required to cross-out all the zeros is 4 which is less than the order of the cost matrix. Hence, we subtract the smallest cost 1 among all uncrossed cost elements from all the uncrossed cost elements, add the smallest cost 1 to the crossing cost elements and leave the others (other cost elements) unchanged. Thus, we obtain the following :

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| I | 4 | 2 | 2 | 2 | 0 |
| II | 0 | 1 | 0 | 2 | 4 |
| III | 4 | 2 | 2 | 0 | 2 |
| IV | 4 | 0 | 0 | 6 | 0 |
| V | 0 | 0 | 0 | 3 | 0 |

Clearly, the minimum number of lines required to cross out all the zeros is 5 which is equal to the order of the cost matrix. Hence, the optimal stage of assignment is reached. We, therefore, start assignment by using box notations as follows:

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| I | 4 | 2 | 2 | 2 | $\boxed{0}$ |
| II | $\boxed{0}$ | 1 | 0 | 2 | 4 |
| III | 4 | 2 | 2 | $\boxed{0}$ | 2 |
| IV | 4 | $\boxed{0}$ | 0 | 6 | $\cancel{0}$ |
| V | 0 | 0 | 0 | 3 | $\cancel{0}$ |

Hence, an optimal assignment is given by

$$I \rightarrow M_5,\ II \rightarrow M_1,\ III \rightarrow M_4,\ IV \rightarrow M_2 \text{ and } V \rightarrow M_3.$$

The cost of the above assignment = Rs. (4 + 5 + 3 + 5 + 6) i.e. Rs. 23.

An alternative optimal assignment is given by :

$$I \rightarrow M_5,\ II \rightarrow M_3,\ III \rightarrow M_4,\ IV \rightarrow M_2 \text{ and } V \rightarrow M_1$$

The cost of this set of assignment = Rs. (4 + 5 + 3 + 5 + 6) i.e. Rs. 23, which is obviously the same as the first one.

**Example 4.** Find the optimal assignment for the following problem :

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| A | 11 | 17 | 8 | 16 | 20 |
| B | 9 | 7 | 12 | 6 | 15 |
| C | 13 | 16 | 15 | 12 | 16 |
| D | 21 | 24 | 17 | 28 | 26 |
| E | 14 | 10 | 12 | 11 | 15 |

**Solution :** The smallest cost elements in the 1st, 2nd, 3rd, 4th and 5th rows are 8, 6, 12, 17 and 10 respectively. Subtracting the smallest cost of each row from all the cost elements of the corresponding row, we get

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| A | 3 | 9 | 0 | 8 | 12 |
| B | 3 | 1 | 6 | 0 | 9 |
| C | 1 | 4 | 3 | 0 | 4 |
| D | 4 | 7 | 0 | 11 | 9 |
| E | 4 | 0 | 2 | 1 | 15 |

Now, subtracting the smallest cost of each column from all the cost elements of the corresponding columns, we get

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| A | 2 | 9 | 0 | 8 | 8 |
| B | 2 | 1 | 6 | 0 | 5 |
| C | 0 | 4 | 3 | 0 | 0 |
| D | 3 | 7 | 0 | 11 | 5 |
| E | 3 | 0 | 2 | 1 | 1 |

Now, we see that the minimum number of horizontal and vertical lines to cross out all the zeros in the above matrix is 4 which is less than the order of the cost matrix.

Hence, we subtract the smallest cost element 1 among the uncrossed cost elements from all uncrossed cost elements and add 1 to the crossing cost elements at the cells (3, 2) i.e. (C, $M_2$), (3,3) i.e. (C, $M_3$), and (3,4) i.e. (C, $M_4$) and leave all others (other cost elements) as they are. Thus, we get

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| A | 1 | 9 | 0 | 8 | 7 |
| B | 1 | 1 | 6 | 0 | 4 |
| C | 0 | 5 | 4 | 1 | 0 |
| D | 2 | 7 | 0 | 1 | 4 |
| E | 2 | 0 | 2 | 4 | 0 |

Again, we see that the number of lines required to cross out all the zeros is 4 which is less than the order 5 of the cost matrix. So, again we subtract the smallest cost element among the uncrossed cost elements from all uncrossed elements, add the same smallest cost element to the crossing cost elements and leave others as they are, to obtain the following:

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| A | $\boxed{0}$ | 8 | $\cancel{0}$ | 8 | 6 |
| B | $\cancel{0}$ | $\cancel{0}$ | 6 | $\boxed{0}$ | 3 |
| C | $\cancel{0}$ | 5 | 5 | 2 | $\boxed{0}$ |
| D | 1 | 6 | $\boxed{0}$ | 1 | 3 |
| E | 2 | $\boxed{0}$ | 3 | 2 | $\cancel{0}$ |

Clearly, the minimum number of horizontal and vertical lines required to cross-out all the zeros is 5 which is equal to the order of the cost matrix. Hence, the optimal stage of assignment is reached.

The optimal assignment is obtained by singling out zeros in rows and columns and then boxing them as shown above and is given as follows :

A ® M1, B ® M4, C ® M5, D ® M3 and E ® M2.

The minimum cost of assignment = Rs. (11 + 6 + 16 + 17 + 1), = Rs. 60.

## EXERCISE

**1.** Find the optimal assignment for the following problem :

| | I | II | III | IV |
|---|---|---|---|---|
| A | 5 | 3 | 1 | 8 |
| B | 7 | 9 | 2 | 6 |
| C | 6 | 4 | 5 | 7 |
| D | 5 | 7 | 7 | 6 |

**2.**Solve the following assignment problem by using Hungarian method :

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | 1 | 4 | 6 | 3 |
| $J_2$ | 9 | 7 | 10 | 9 |
| $J_3$ | 4 | 5 | 11 | 7 |
| $J_4$ | 8 | 7 | 28 | 5 |

**3.** Solve the following assignment problem by using Hungarian method :

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| I | 12 | 30 | 21 | 15 |
| II | 18 | 33 | 9 | 31 |
| III | 44 | 25 | 24 | 21 |
| IV | 23 | 30 | 28 | 14 |

**4.** Solve the following assignment problem by using Hungarian method :

| | I | II | III | IV |
|---|---|---|---|---|
| A | 10 | 12 | 9 | 11 |
| B | 5 | 10 | 7 | 8 |
| C | 12 | 14 | 13 | 11 |
| D | 8 | 15 | 11 | 9 |

**5.** Solve the following cost minimization problem by using Hungarian method :

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 11 | 10 | 18 | 5 | 9 |
| B | 14 | 13 | 12 | 19 | 6 |
| C | 5 | 3 | 4 | 2 | 4 |
| D | 15 | 18 | 17 | 9 | 12 |
| E | 10 | 11 | 19 | 6 | 14 |

## ANSWERS

**1.** A → III, B → IV, C → II, D → I; Minimum Cost = 16 units.

**2.** $J_1 \rightarrow M_1$, $J_2 \rightarrow M_3$, $J_3 \rightarrow M_2$, $J_4 \rightarrow M_4$; Minimum Cost = 21 units.

**3.** I → 1, II → 3, III → 2, IV → 4; Minimum Cost = 60 units.

**4.** A → III, B → I, C → II, D → IV; Minimum Cost = 37 units.

**5.** A→II, B→V, C→III, D→IV, E→I; Minimum Cost = 39 units.

## Some Special Types of Assignment Problems

The following special types of assignment problems are of great importance in the subject :

### Maximization Assignment Problem

All the assignment problems dealt with so far are cost-minimization problems but sometimes assignment problems also exist with profit maximization objective. The method of solving such problems is not a new one but a simple modification of the method of solving cost minimization assignment problems. Indeed, if all the cost-elements are replaced by the elements obtained by subtracting the cost elements from a large constant, say, the largest of all the cost elements, then the problem is reduced to a minimization problem and solution to this problem will be the solution to original assignment problem.

The following example will illustrate the method of solution of maximization assignment problem :

**Example 1.** A company has 5 jobs to be done. The following matrix shows the return (in rupees) of assigning the $i^{th}$ machine to the $j^{th}$ job, ($i$ and $j$ =1, 2, 3, 4, 5) :

$$\begin{bmatrix} 9 & 3 & 4 & 2 & 10 \\ 12 & 10 & 8 & 11 & 9 \\ 11 & 2 & 9 & 0 & 8 \\ 8 & 0 & 10 & 3 & 7 \\ 7 & 5 & 6 & 2 & 9 \end{bmatrix}$$

Assign the jobs so as to maximize the total expected profit.

**Solution :** We see that the largest cost element within the cost elements of the given matrix is 12. Hence, subtracting each cost element of the cost matrix from 12 and writing in their corresponding places, we get the following reduced problem in minimization form :

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 3 | 9 | 8 | 10 | 2 |
| B | 0 | 2 | 4 | 1 | 3 |
| C | 1 | 10 | 3 | 12 | 4 |
| D | 4 | 12 | 2 | 9 | 5 |
| E | 5 | 7 | 6 | 10 | 3 |

The smallest cost elements in the 1st, 2nd, 3rd, 4th and 5th rows are 2, 0, 1, 2 and 3 respectively.

Subtracting the smallest cost element of each row from all the cost elements of the corresponding rows, we get

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 1 | 7 | 6 | 8 | 0 |
| B | 0 | 2 | 4 | 1 | 3 |
| C | 0 | 9 | 2 | 11 | 3 |
| D | 2 | 10 | 0 | 7 | 3 |
| E | 2 | 4 | 3 | 7 | 0 |

Again, subtracting the smallest cost element of each column from all the cost elements of the corresponding columns, we get

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 1 | 5 | 6 | 7 | 0 |
| B | 0 | 0 | 4 | 0 | 3 |
| C | 0 | 7 | 2 | 10 | 3 |
| D | 2 | 8 | 0 | 6 | 3 |
| E | 2 | 2 | 3 | 6 | 0 |

Evidently, the number of horizontal and vertical lines to cross out all zeros is 4 which is less than the order of the cost matrix. So, we improve the solution by subtracting the smallest cost element of the uncrossed cost elements from all the uncrossed cost elements and by adding that to the crossing cost elements and thus, we get

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 1 | 3 | 6 | 5 | $\boxed{0}$ |
| B | 2 | $\cancel{0}$ | 5 | $\boxed{0}$ | 5 |
| C | $\boxed{0}$ | 5 | 2 | 8 | 3 |
| D | 2 | 6 | $\boxed{0}$ | 4 | 3 |
| E | 2 | $\boxed{0}$ | 3 | 4 | $\cancel{0}$ |

Now, we see that the number of horizontal and vertical lines to croos-out all zeros is 5, which is equal to the order of the cost matrix. Hence, the optimal stage of assignment is reached. To get the optimal solution, we start boxing as shown above. Thus, the optimal solution is given by :

$$A \rightarrow V, B \rightarrow IV, C \rightarrow I, D \rightarrow III, E \rightarrow II.$$

Using the original matrix given in the problem, the maximum total expected profit Z is given by Z = Rs. (10 + 11 + 11 + 10 + 5) = Rs. 47.

## Restrictive Assignment Problem

Such assignment problems are not rare where due to technical reasons or socio-legal reasons assignments are restricted. This type of assignment problems is tackled by assigning a very high cost to the corresponding cells so that the activity is automatically excluded from the optimal solution.

The following example will illustrate the method of solution of restrictive (or restricted) assignment problem :

**Example 2.** Four new machines $M_1$, $M_2$, $M_3$ and $M_4$ are to be installed in a machine shop. There are four vacant places A,B,C and D available. Because of limited waiting space, $M_2$ cannot be placed at C and $M_3$ cannot be placed at A. The assignment costs in rupees of installing the $i^{th}$ machine at the $j^{th}$ place are given as follows :

| | A | B | C | D |
|---|---|---|---|---|
| $M_1$ | 9 | 11 | 15 | 10 |
| $M_2$ | 12 | 9 | – | 10 |
| $M_3$ | – | 11 | 14 | 11 |
| $M_4$ | 14 | 8 | 15 | 7 |

where the cells ($M_2$, C) and ($M_3$, A) correspond to the restricted assignments.

Obtain an optimal solution of the above problem which gives minimization of total cost.

**Solution :** We write the cost matrix either writing $\infty$ or the largest of the cost elements in the cells which correspond to the restricted assignments as follows :

| | A | B | C | D |
|---|---|---|---|---|
| $M_1$ | 9 | 11 | 15 | 10 |
| $M_2$ | 12 | 9 | 15 | 10 |
| $M_3$ | 15 | 11 | 14 | 11 |
| $M_4$ | 14 | 8 | 12 | 7 |

The smallest cost elements in the $1^{st}$, $2^{nd}$, $3^{rd}$ and $4^{th}$ rows are 9, 9, 11 and 7 respectively.

Subtracting the smallest cost element of each row from all the cost elements of the corresponding rows, we get

| | A | B | C | D |
|---|---|---|---|---|
| $M_1$ | 0 | 2 | 6 | 1 |
| $M_2$ | 3 | 0 | 6 | 1 |
| $M_3$ | 4 | 0 | 3 | 0 |
| $M_4$ | 7 | 1 | 5 | 0 |

Next, subtracting the smallest cost element of each column from all the cost elements of the corresponding columns, we get

| | A | B | C | D |
|---|---|---|---|---|
| $M_1$ | $\boxed{0}$ | 2 | 6 | 1 |
| $M_2$ | 3 | $\boxed{0}$ | 6 | 1 |
| $M_3$ | 4 | 0 | $\boxed{0}$ | 0 |
| $M_4$ | 7 | 1 | 5 | $\boxed{0}$ |

Clearly, the minimum number of horizontal and vertical lines required to cross out all zeros is 4which equals the order of the cost matrix, therefore, the optimal stage of assignment is reached. We, therefore, start boxing as shown above and obtain the solution as :

$$M_1 \rightarrow A,\ M_2 \rightarrow B,\ M_3 \rightarrow C,\ M_4 \rightarrow D$$

The minimum assignment cost = Rs. (9 + 9 + 14 + 7) = Rs. 39

## Unbalanced Assignment Problem

Some assignment problems may be unbalanced i.e. the number of machines may be different from the number of jobs. Such problems are solved by introducing hypothetical i.e. dummy jobs or machines as the case may be. The following example will make the technique clear :

**Example 3.** A truck company on a particular day has 4 trucks for sending materials to 6 terminals. The cost of sending materials to the terminals by the trucks is given below.

Trucks

| | | A | B | C | D |
|---|---|---|---|---|---|
| Terminals | 1 | 3 | 6 | 2 | 6 |
| | 2 | 7 | 1 | 4 | 4 |
| | 3 | 3 | 8 | 5 | 8 |
| | 4 | 6 | 4 | 3 | 7 |
| | 5 | 5 | 2 | 4 | 4 |
| | 6 | 5 | 7 | 6 | 2 |

Find how the cost can be minimized.

**Solution :**Since, the number of terminals is greater than the number of trucks by 2, therefore, we rewrite the cost matrix with zero cost as the cost elements for hypothetical trucks as follows :

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 3 | 6 | 2 | 6 | 0 | 0 |
| 2 | 7 | 1 | 4 | 4 | 0 | 0 |
| 3 | 3 | 8 | 5 | 8 | 0 | 0 |
| 4 | 6 | 4 | 3 | 7 | 0 | 0 |
| 5 | 5 | 2 | 4 | 4 | 0 | 0 |
| 6 | 5 | 7 | 6 | 2 | 0 | 0 |

Subtracting the smallest cost element of each row from all the cost elements of the corresponding rows, we get the same matrix as above. Then, subtracting the smallest cost element of each column from all the cost elements of the corresponding columns, we get

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 0 | 5 | $\boxed{0}$ | 4 | $\cancel{0}$ | $\cancel{0}$ |
| 2 | 4 | $\boxed{0}$ | 2 | 2 | $\cancel{0}$ | $\cancel{0}$ |
| 3 | $\boxed{0}$ | 7 | 3 | 6 | $\cancel{0}$ | $\cancel{0}$ |
| 4 | 3 | 3 | 1 | 5 | $\boxed{0}$ | $\cancel{0}$ |
| 5 | 2 | 1 | 2 | 2 | $\cancel{0}$ | $\boxed{0}$ |
| 6 | 2 | 6 | 4 | $\boxed{0}$ | $\cancel{0}$ | $\cancel{0}$ |

Clearly, the minimum number of horizontal and vertical lines required to cross-out all zeros is 6 which equals the order of the cost matrix. Hence, the optimal stage of assignment is reached and we start boxing as above to obtain the optimal solution as follows :

$1 \to C$, $2 \to B$, $3 \to A$, $4 \to E$ (hypothetical), $5 \to F$ (hypothetical), $6 \to D$

or simply $1 \to C$, $2 \to B$, $3 \to A$, $6 \to D$,

where the terminals 4 and 5 are to remain free from any assignment.

The corresponding minimum cost = Rs. (2 + 1 + 3 + 2) = Rs. 8.

# Travelling Salesman Problem

A typical problem known as the travelling salesman problem concerns the travelling of a salesman from one destination to another to cover all the destinations and back to the starting point. The problem is to determine a path which a salesman must choose so as to cover the minimum distance. Thus, the cost matrix in a travelling salesman problem is made of the distances between different destinations and, therefore, is a rectangular matrix. As a convention, the distance between a destination to itself is taken as infinity.

A solution requires the determination of a series of destinations starting and ending at the same place and covering the minimum distance. It may be observed that there is some analogy of this problem to the assignment problem and the only difference that distinguishes a travelling salesman problem is that a solution obtained by the standard techniques of solving assignment problems may not conform to the requirements of the travelling salesman problem. So, for a solution of a travelling salesman problem, we invoke the methods of solving assignment problems but modify that to get the desired solution. In fact, there are not too many methods of solving travelling salesman problems. The ***branch and boundtechnique*** is a quite general one but because of the difficulty level associated with this, we shall consider here only the ***trial and errormethod*** over the solution obtained by treating it as an assignment problem.

The ***trial and errormethod*** begins with obtaining an optional solution, treating the problem as an assignment problem.

**Note :**The associated matrix is, in general, symmetric with ∞ at the leading diagonal but the method can be applied even when the matrix is not symmetric.

If the solution does not provide a closed path, i.e. a path which starts and ends at the same place, traversing each destination just once, then we modify the path by considering the next minimum distance, if necessary.

The following example will illustrate the method of solution of travelling salesman problem :

**Example 4.**A salesman has to visit five cities A,B,C,D and E. The distance (in hundred km) between every pair of cities beings given in the following table as a matrix. If the salesman starts from city A and has to come back to that city after covering all the cities once only, which route should he follow so that the total distance travelled is minimum.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | $\infty$ | 7 | 6 | 8 | 4 |
| B | 7 | $\infty$ | 8 | 5 | 6 |
| C | 6 | 8 | $\infty$ | 9 | 7 |
| D | 8 | 5 | 9 | $\infty$ | 8 |
| E | 4 | 6 | 7 | 8 | $\infty$ |

**Solution :** We first obtain a solution treating the above as an assignment problem. To this end, we subtract the smallest cost element of each row from all the cost elements of the corresponding rows, we get

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | $\infty$ | 3 | 2 | 4 | 0 |
| B | 2 | $\infty$ | 3 | 0 | 1 |
| C | 0 | 2 | $\infty$ | 3 | 1 |
| D | 3 | 0 | 4 | $\infty$ | 3 |
| E | 0 | 2 | 3 | 4 | $\infty$ |

Again, subtracting the smallest cost element of each column from all the cost elements of the corresponding columns, we get

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | ∞ | 3 | 0 | 4 | 0 |
| B | 2 | ∞ | 1 | 0 | 1 |
| C | 0 | 2 | ∞ | 3 | 1 |
| D | 3 | 0 | 2 | ∞ | 3 |
| E | 0 | 2 | 1 | 4 | ∞ |

We now see that all the zeros are cross-out by drawing four lines (2 horizontal and 2 vertical) as shown in the above table which is less than the order of the cost matrix. So, subtracting the smallest cost element of the uncrossed cost elements from the uncrossed cost elements and also adding the smallest cost element to the crossing cost elements at the cells (A, A), (A, B), (B, A), (B, B) and leaving all other cost elements as they are, we get

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | ∞ | 4 | 0 | 4 | 0 |
| B | 3 | ∞ | 1 | 0 | 1 |
| C | 0 | 2 | ∞ | 2 | 0 |
| D | 3 | 0 | 1 | ∞ | 2 |
| E | 0 | 2 | 0 | 3 | ∞ |

Evidently, five horizontal and vertical or either of these two lines are necessary to cross-out all the zeros which equals to the order of the cost matrix. Now, we see as a solution one gets the following :

$B \to D \to B$, $A \to C \to E \to A$, $A \to E \to C \to A$

Here, we observe that if the salesman starts from B, he can cover only the town D with the minimum cost and returns to B without covering the towns A, C and E.

Again, we observe that if the salesman starts from A, he can cover only the towns C and E in both ways i.e. first C and then E or first E and then C with the minimum cost and returns to A without covering the towns B and D. Thus, none of these tours are acceptable.

So, we look for a route which does not go by the smallest requirement zero in the above reduced matrix but next-to-smallest is considered. Clearly, this smallest requirement cost 1 occurs in the cells (2, 3), (2, 5) and (4, 3). But, even with the element 1, there is no route connecting A to the rest of the towns. So, with the next smallest requirement cost element i.e. 2, we try to find out a complete route as follows :

From A, we move to C; from C, we move to B to D; from D to E and finally, from E to A, i.e.

$$A \rightarrow C \rightarrow B \rightarrow D \rightarrow E \rightarrow A.$$

The total distance traversed in the route calculated from the original table is (6 + 8 + 5 + 8 + 4) hundred km, i.e. 3100 km.

Clearly, an alternative complete route is A → C → B → D → E → A and the corresponding total distance is 3100 km.

**Remark :** A travelling salesman problem need not always have only distance as the determinable parameter, time may also matter and both time and distance need not be the same for one can get a non-symmetric matrix as the following problem poses.

**Example 5.** A medical representative has to visit four doctors B,C,D and E. He does not want to visit any doctor twice after completing tour of all doctors, he wishes to return to his own office starting his tour from his office only. An appropriate time in hours of moving from one doctor to another including his office is given in the following table. Determine a route so that he is back to his office in the quickest possible time.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 1 | 4 | 7 | 1 |
| B | 3 | – | 2 | 7 | 2 |
| C | 8 | 6 | – | 4 | 6 |
| D | 9 | 3 | 5 | – | 7 |
| E | 1 | 2 | 2 | 7 | – |

**Solution :** We proceed to find an optimal solution as per methods available for solving assignment problems and to this end, we subtract the smallest cost element of each row from all the cost elements of the corresponding rows and obtain the following :

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 0 | 3 | 6 | 0 |
| B | 1 | – | 0 | 5 | 0 |
| C | 4 | 2 | – | 0 | 2 |
| D | 6 | 0 | 2 | – | 4 |
| E | 0 | 1 | 1 | 6 | – |

Again, subtracting the smallest cost element of each column from all the cost elements of the corresponding columns, we get

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 0 | 3 | 6 | $\boxed{0}$ |
| B | 1 | – | $\boxed{0}$ | 5 | 0 |
| C | 4 | 2 | – | $\boxed{0}$ | 2 |
| D | 6 | 0 | 2 | – | 4 |
| E | $\boxed{0}$ | 1 | 1 | 6 | 1 |

Clearly, five lines (horizontal and vertical) are required to cross-out all the zeros which equals to the order of the cost matrix. Therefore, from the above, we get the solution as :

$$A \rightarrow E \rightarrow A, \qquad B \rightarrow C \rightarrow D \rightarrow B$$

In order that the route is complete, one can choose the following route considering the next to smallest cost elements of the above table as :

$A \rightarrow E \rightarrow B \rightarrow C \rightarrow D \rightarrow A$ or $A \rightarrow E \rightarrow C \rightarrow D \rightarrow B \rightarrow A$

with respect to total time (1 + 2 + 2 + 4 + 9) hours i.e. 18 hours and (1 + 2 + 4 + 3 + 3) hours i.e. 13 hours. Evidently, the optional route will be

$A \rightarrow E \rightarrow C \rightarrow D \rightarrow B \rightarrow A$ with the quickest possible time of 13 hours.

As a concluding remark, we mention that :

**(i)**an assignment problem possesses an optional solution and

**(ii)**there may exist many optimal solutions also.

## Flow Chart of Hungarian Method for Solving an Assignment Problem

The steps of Hungarian method for solving an assignment problem can be described by a flow chart as shown in Fig.1 below :

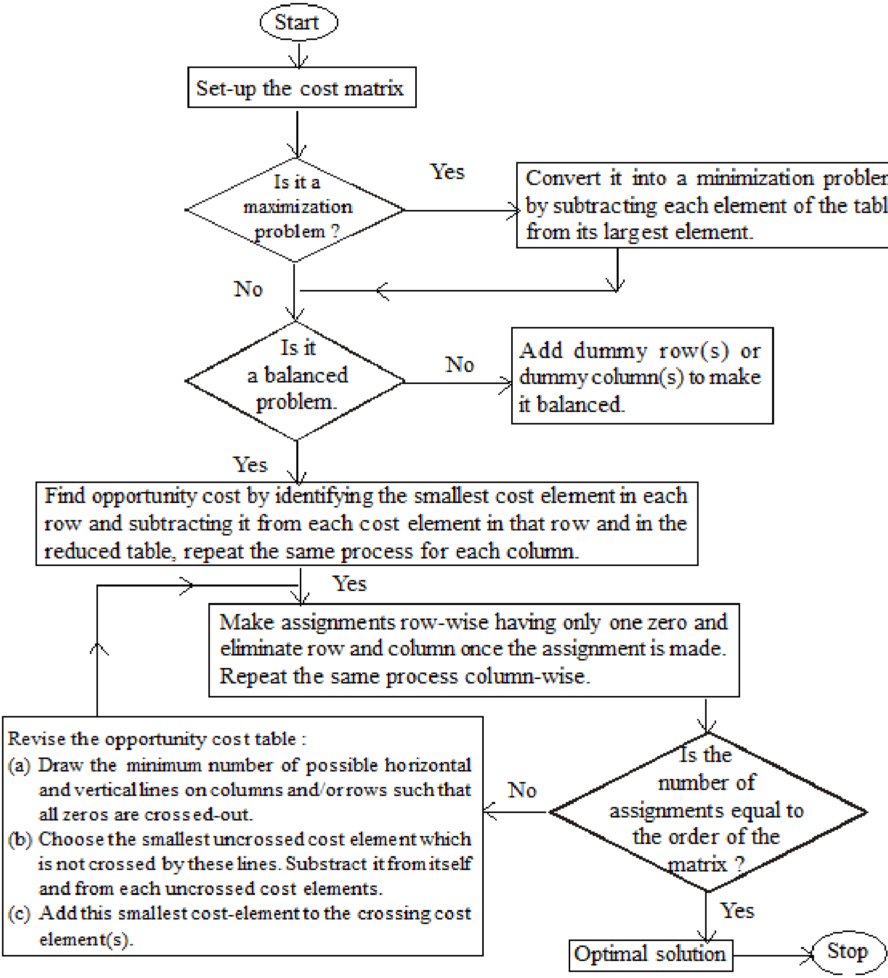


**Fig 1 : Flow Chart of Hungarian Method for Solving an Assignment Problem**

# EXERCISE

**1.** A company has 5 jobs to the done. The following matrix shows the return in rupees on assigning the $i^{th}$ machine to the $j^{th}$job :

| | | Jobs | | | | |
|---|---|---|---|---|---|---|
| | | A | B | C | D | E |
| Machines | I | 5 | 11 | 10 | 12 | 4 |
| | II | 2 | 4 | 6 | 3 | 5 |
| | III | 3 | 12 | 5 | 14 | 6 |
| | IV | 6 | 14 | 4 | 11 | 7 |
| | V | 7 | 9 | 8 | 12 | 5 |

Assign the five jobs to the five machines so as to maximize the total expected profit.

**2.** Five salesmen are to be assigned to five territories. Based on the past performance, the following table shows the annual sales (in lakh rupees) that can be generated by each salesman in each territory :

| | | Territories | | | | |
|---|---|---|---|---|---|---|
| | | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ |
| Salesmen | $S_1$ | 26 | 14 | 10 | 12 | 9 |
| | $S_2$ | 31 | 27 | 30 | 14 | 16 |
| | $S_3$ | 15 | 18 | 16 | 25 | 30 |
| | $S_4$ | 17 | 12 | 21 | 30 | 25 |
| | $S_5$ | 20 | 19 | 25 | 16 | 10 |

Find the optimal assignment of the above problem.

**3.** Five operators have to be assigned to five machines. The assignment costs are given in the following table :

Machines

| | | I | II | III | IV | V |
|---|---|---|---|---|---|---|
| Operators | A | 5 | 5 | – | 2 | 6 |
| | B | 7 | 4 | 2 | 3 | 4 |
| | C | 9 | 3 | 5 | – | 3 |
| | D | 7 | 2 | 6 | 7 | 2 |
| | E | 6 | 5 | 7 | 9 | 1 |

Operator A cannot operate machine III and operator C cannot operate
machine IV. Find the optimal assignment schedule.

**4.** Four operators $O_1$, $O_2$, $O_3$ and $O_4$ are available to a manager who has to get four jobs $J_1$, $J_2$, $J_3$ and $J_4$ done by assigning one job to each operator. The times in minutes needed by different operators for different jobs are given in the following matrix :

| | A | B | C | D |
|---|---|---|---|---|
| $O_1$ | 12 | 10 | 10 | 8 |
| $O_2$ | 14 | 12 | 15 | 11 |
| $O_3$ | 6 | 10 | 16 | 4 |
| $O_4$ | 8 | 10 | 9 | 7 |

**(i)** How should the manager assign the jobs to the operators so that the total time needed for all the four jobs is minimum ?

**(ii)** If job $J_2$ is not be assigned to the operator $O_2$, then what should be the assignment and how much additional total time will be required ?

**5.** In a machine shop, a supervisor wishes to assign five jobs among six machines. Any one of the jobs can be processed completely by any one of the machines as given below :

| | | Machines | | | | | |
|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | F |
| Jobs | 1 | 13 | 13 | 16 | 23 | 19 | 9 |
| | 2 | 11 | 19 | 26 | 16 | 17 | 18 |
| | 3 | 12 | 11 | 4 | 9 | 6 | 10 |
| | 4 | 7 | 15 | 9 | 14 | 14 | 13 |
| | 5 | 9 | 13 | 12 | 8 | 14 | 11 |

The assignment of jobs to machines be on a one-to-one basis. Assign the jobs to machines so that the total cost is minimum. Find the minimum total cost.

**6.** Solve the following unbalanced assignment problem of minimizing total time for doing all the jobs :

| | | Jobs | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| Operators | $O_1$ | 6 | 2 | 5 | 2 | 6 |
| | $O_2$ | 2 | 5 | 8 | 7 | 7 |
| | $O_3$ | 7 | 8 | 6 | 9 | 8 |
| | $O_4$ | 6 | 2 | 3 | 4 | 5 |
| | $O_5$ | 9 | 3 | 8 | 9 | 7 |
| | $O_6$ | 4 | 7 | 4 | 6 | 8 |

7. A salesman must travel from city to city to maintain his accounts. This week he has to leave his home base and visit each other city and return home. The following table shows the distances (in kilometers) between the various cities. The home city is city A. Use the assignment technique to determine the tour that will minimize the total distance of visiting all cities and return home :

To Cities

| From Cities | A | B | C | D | E |
|---|---|---|---|---|---|
| A | – | 375 | 600 | 150 | 190 |
| B | 375 | – | 300 | 350 | 175 |
| C | 600 | 300 | – | 350 | 500 |
| D | 160 | 350 | 350 | – | 300 |
| E | 190 | 175 | 500 | 300 | – |

**8.** A machine operator processes five types of items on his machine each week and must choose a sequence for them. The set-up cost per change depends on the item presently on the machine and the set-up to be made according to the following table :

No Items

| From Items | A | B | C | D | E |
|---|---|---|---|---|---|
| A | $\infty$ | 4 | 7 | 3 | 4 |
| B | 4 | $\infty$ | 6 | 3 | 4 |
| C | 7 | 6 | $\infty$ | 7 | 5 |
| D | 3 | 3 | 7 | $\infty$ | 7 |
| E | 4 | 4 | 5 | 7 | $\infty$ |

If he processes each type of item once and only once each week, then how should he sequence the items on his machine in order to minimize the total set-up cost ?

**9.** Write the following assignment problem as a transportation problem :

| | | Jobs I | Jobs II | Jobs III |
|---|---|---|---|---|
| | 2 | 3 | 5 | 7 |
| Workers | 2 | 6 | 4 | 8 |
| | 3 | 7 | 5 | – |

where (-) indicates that worker 3 cannot perform job III.

**10.** Write the following assignment problem as a linear programming problem :

| | | Jobs $J_1$ | Jobs $J_2$ | Jobs $J_3$ |
|---|---|---|---|---|
| | $M_1$ | 8 | 7 | 6 |
| Machines | $M_2$ | 5 | 7 | 8 |
| | $M_3$ | 6 | 8 | 7 |

## ANSWERS

**1.** I→3, II→5, III→4, IV→2, V→1; Maximum Total Profit = Rs. 50.

**2.** $S_1 \to T_1$, $S_2 \to T_2$, $S_3 \to T_5$, $S_4 \to T_4$, $S_5 \to T_3$; Maximum Sales Generated = 138 lakh rupees.

**3.** A→IV, B→III, C→II, D→I, E→V or A→IV, B→III, C→V, D→II, E→I; Minimum Cost = 15 units.

**4.** (i) $O_1 \to J_3$, $O_2 \to J_2$, $O_3 \to J_4$, $O_4 \to J_1$; Minimum Time = 34 minutes.

(ii) $O_1 \to J_2$, $O_2 \to J_4$, $O_3 \to J_1$, $O_4 \to J_3$, Minimum Time = 36 minutes.

Additional Time Required = (36 – 34) minutes = 2 minutes.

**5.** 1→F, 2→A, 3→E, 4→C, 5→D; Minimum Total Cost = 43 units.

**6.** $O_1 \to 4$, $O_2 \to 1$, $O_3 \to$ hypothetical 6, $O_4 \to 5$, $O_5 \to 2$, $O_6 \to 3$; Minimum Time = 16 units.

**7.** $A \to D$, $B \to E$, $C \to B$, $D \to C$, $E \to A$; Minimum Distance = 1165 km.

**8.** $A \to D$, $B \to A$, $C \to E$, $D \to B$, $E \to C$; Minimum Set-up Cost = 20 units.

**9.**

| Jobs→ ↓ Workers | I | II | III | Supply |
|---|---|---|---|---|
| 1 | 3 | 5 | 7 | 1 |
| 2 | 6 | 4 | 8 | 1 |
| 3 | 7 | 5 | M | 1 |
| Demand | 1 | 1 | 1 | 3 |

where M is very high cost.

**10.** Minimize $Z = (8x_{11} + 7x_{12} + 6x_{13}) + (5x_{21} + 7x_{22} + 8x_{23}) + (6x_{31} + 8x_{32} + 7x_{33})$

subject to the following constraints :

$$x_{i1} + x_{i2} + x_{i3} = 1;\ i = 1, 2, 3,$$

$$x_{1j} + x_{2j} + x_{3j} = 1;\ j = 1, 2, 3$$

and satisfying $x_{ij} \geq 0$; $i$ and $j$ =1, 2, 3,

where $x_{ij} = \begin{cases} 1, \text{if } i^{th} \text{machine processes the } j^{th} \text{job} \\ 0\ ; \text{otherwise} \end{cases}$

## Sensitivity in Assignment Problems

The structure of assignment problems is of such a type that there is a very little scope for sensitivity analysis. Modest alterations in the conditions such as one being able to do two jobs can be considered by repeating the worker's row and adding a hypothetical (i.e. dummy) column to square up the matrix (i.e. to balance the assignment problem). Addition of a fixed constant throughout anyrow or any column also makes no difference to the schedule of optimal assignment. However, sometimes, equi-proportionate change throughout a row or a column can make a difference. So, with reference to the assignment problem , there is no scope for altering the level (schedule) of assignment.

## MISCELLANEOUS SOLVED EXAMPLES

**Example :**A company has four employees and four fixed jobs. Each job can be assigned to one and only one employee. The number of hours each employee would take to perform each job is as follows :

| | A | B | C | D |
|---|---|---|---|---|
| I | 42 | 35 | 28 | 21 |
| II | 30 | 25 | 20 | 15 |
| III | 30 | 25 | 20 | 15 |
| IV | 24 | 20 | 16 | 12 |

Find an optimal assignment and also the minimum time in hours.

**Solution :** The smallest cost elements in the 1st, 2nd, 3rd and 4th rows are 21, 15, 15 and 12 respectively. Subtracting the smallest cost element of each row from the cost elements of the corresponding rows, we get.

| | A | B | C | D |
|---|---|---|---|---|
| I | 21 | 14 | 7 | 0 |
| II | 15 | 10 | 5 | 0 |
| III | 15 | 10 | 5 | 0 |
| IV | 12 | 8 | 4 | 0 |

Again, subtracting the smallest cost element of each column from the cost elements of the corresponding columns, we get

| | A | B | C | D |
|---|---|---|---|---|
| I | 9 | 6 | 3 | 0 |
| II | 3 | 2 | 1 | 0 |
| III | 3 | 2 | 1 | 0 |
| IV | 0 | 0 | 0 | 0 |

Now, the minimum number of horizontal and vertical lines to cross-out all the zeros is 2 which is less than the order of the cost matrix. So, subtracting the smallest cost element of the uncrossed cost elements from all of the uncrossed cost elements and adding the same to the crossing cost element and leaving all other cost elements, as they are, we get :

| | A | B | C | D |
|---|---|---|---|---|
| I | 8 | 5 | 2 | 0 |
| II | 2 | 1 | 0 | 0 |
| III | 2 | 1 | 0 | 0 |
| IV | 0 | 0 | 0 | 1 |

Again, the minimum number of horizontal and vertical lines required to cross-out all zeros is 3, which is again less than the order of the cost matrix. So, following the same procedure, we improve the situation and obtain the following :

| | A | B | C | D |
|---|---|---|---|---|
| I | 7 | 4 | 2 | $\boxed{0}$ |
| II | 1 | $\boxed{0}$ | $\cancel{0}$ | $\cancel{0}$ |
| III | 1 | $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ |
| IV | $\boxed{0}$ | $\cancel{0}$ | $\cancel{0}$ | 2 |

Clearly, the minimum number of horizontal and vertical lines required to cross-out all the zeros is 4 which is equal to the order of the cost matrix. Hence, the optimal stage of assignment is reached. We, therefore, start boxing the zeros which occur singularly in a row or in a column and then make choices for more than one zeros occurring in a row or column. Thus, the solution i.e. the optimal assignment are :

(i) I $\rightarrow$ D, II $\rightarrow$ B, III $\rightarrow$ C, IV $\rightarrow$ A; the minimum time = (21 + 25 + 20 + 24) = 90 hours.

(ii)I $\rightarrow$ D, II $\rightarrow$ C, III $\rightarrow$ B, IV $\rightarrow$ A; the minimum time = (21 + 20 + 25 + 24) = 90 hours.

$\therefore$ The minimum time of assignment is 90 hours.

**Example 2.** A company has five employees A,B,C,D and E and five fixed jobs U,V,X,Y and Z. The number of hours each employee would take to perform each job is as follows :

| | A | B | C | D | E |
|---|---|---|---|---|---|
| U | 3 | 5 | 10 | 15 | 8 |
| V | 4 | 7 | 15 | 18 | 8 |
| X | 8 | 15 | 20 | 20 | 12 |
| Y | 5 | 5 | 8 | 10 | 6 |
| Z | 10 | 10 | 15 | 25 | 10 |

**Solution :**The smallest cost elements in the 1$^{st}$, 2$^{nd}$, 3$^{rd}$, 4$^{th}$ and 5$^{th}$ rows are 3, 4, 8, 5 and 10. Subtracting the smallest cost element of each row from the cost elements of the corresponding rows, we get

| | A | B | C | D | E |
|---|---|---|---|---|---|
| U | 0 | 2 | 7 | 12 | 5 |
| V | 0 | 3 | 11 | 14 | 4 |
| X | 0 | 4 | 12 | 12 | 4 |
| Y | 0 | 0 | 3 | 5 | 1 |
| Z | 0 | 0 | 5 | 15 | 0 |

Again, subtracting the smallest cost element of each column from the cost elements of the corresponding columns, we get

| | A | B | C | D | E |
|---|---|---|---|---|---|
| U | 0 | 2 | 4 | 7 | 5 |
| V | 0 | 3 | 8 | 9 | 4 |
| X | 0 | 4 | 9 | 7 | 4 |
| Y | 0 | 0 | 0 | 0 | 1 |
| Z | 0 | 0 | 2 | 10 | 0 |

Now, we see that the minimum number of horizontal and vertical lines to cross-out all the zeros is 3 which **.,/**is less than the order of the cost matrix. So, we subtract the smallest cost element of the uncrossed cost elements from all the uncrossed cost elements and add the same to the crossing cost elements and leave all other cost elements as they are. Thus, we obtain the following :

| | A | B | C | D | E |
|---|---|---|---|---|---|
| U | 0 | 0 | 2 | 5 | 3 |
| V | 0 | 1 | 6 | 7 | 2 |
| X | 0 | 2 | 7 | 5 | 2 |
| Y | 2 | 0 | 0 | 0 | 1 |
| Z | 2 | 0 | 2 | 10 | 0 |

Again, the minimum number of horizontal and vertical lines to cross-out all the zeros is 4 which is less than the order of the cost matrix. So, we proceed further to obtain :

| | A | B | C | D | E |
|---|---|---|---|---|---|
| U | 1 | $\cancel{0}$ | $\boxed{0}$ | 3 | 2 |
| V | $\cancel{0}$ | $\boxed{0}$ | 3 | 4 | $\cancel{0}$ |
| X | $\boxed{0}$ | 1 | 4 | 2 | $\cancel{0}$ |
| Y | 5 | 2 | $\cancel{0}$ | $\boxed{0}$ | 2 |
| Z | 4 | 1 | 1 | 9 | $\boxed{0}$ |

Clearly, the minimum number of horizontal and vertical lines required to cross-out all the zeros is 5 which equals the order of the cost matrix. Hence, the optimal stage of assignment is reached. We, therefore, identify the zeros singularly in each column and row and then by selection, we obtain the assignment as :

$$U \to C,\ V \to B,\ X \to A,\ Y \to D,\ Z \to E$$

$\therefore$ The minimum time of assignment = (10 + 7 + 8 + 10 + 10) hours = 45 hours.

**Example 3.** Find the optimal assignment and the corresponding assignment cost from the following :

Machines

| | A | B | C | D |
|---|---|---|---|---|
| I | 5 | 3 | 1 | 8 |
| II | 7 | 9 | 2 | 6 |
| III | 6 | 4 | 5 | 7 |
| IV | 5 | 7 | 7 | 6 |

**Solution :** The smallest cost elements in the 1st, 2nd, 3rd and 4th rows are 1, 2, 4 and 5 respectively. Subtracting the smallest cost element of each row from the cost elements of the corresponding rows, we get

| | A | B | C | D |
|---|---|---|---|---|
| I | 4 | 2 | 0 | 7 |
| II | 5 | 7 | 0 | 4 |
| III | 2 | 0 | 1 | 3 |
| IV | 0 | 2 | 2 | 1 |

Again, subtracting the smallest cost element of each column from the cost elements of the corresponding columns, we have

| | A | B | C | D |
|---|---|---|---|---|
| I | 4 | 2 | 0 | 7 |
| II | 5 | 7 | 0 | 3 |
| III | 2 | 0 | 1 | 2 |
| IV | 0 | 2 | 2 | 0 |

Clearly, the minimum number of horizontal and vertical lines to cross-out all the zeros is 3 which is less than the order of the cost matrix. Hence, we improve the solution by subtracting the smallest cost element of the uncrossed cost elements from all the uncrossed cost elements and adding the same to the crossing cost elements and leaving all other cost elements, as they are, we get

| | A | B | C | D |
|---|---|---|---|---|
| I | 2 | 2 | 0 | 5 |
| II | 3 | 7 | 0 | 1 |
| III | 0 | 0 | 1 | 0 |
| IV | 0 | 4 | 4 | 0 |

Again, the number of horizontal and vertical lines to cross-out all the zeros is 3 which is less than 4, the order of the cost matrix. So, we further improve the solution given below :

| | A | B | C | D |
|---|---|---|---|---|
| I | 1 | 1 | $\boxed{0}$ | 4 |
| II | 2 | 6 | $\cancel{0}$ | $\boxed{0}$ |
| III | $\cancel{0}$ | $\boxed{0}$ | 2 | $\cancel{0}$ |
| IV | $\boxed{0}$ | 4 | 5 | 1 |

The minimum number of horizontal and vertical lines to cross-out all the zeros is now 4, which equals the order of the cost matrix. Hence, the optimal stage of assignment is reached. We, therefore, put boxes round the zeros occurring along in a row or column and the remaining by choice and get the solution as :

$$I \rightarrow C,\ II \rightarrow D,\ III \rightarrow B,\ IV \rightarrow A.$$

$\therefore$ The corresponding assignment cost = Rs. (1 + 6 + 4 + 4 + 5) = Rs. 16.

**Example 4.** A company has three senior executives. Each is judged against each of three positions and their ratings are given below :

| | | Positions I | II | III |
|---|---|---|---|---|
| Executives | $E_1$ | 7 | 5 | 6 |
| | $E_2$ | 8 | 4 | 7 |
| | $E_3$ | 9 | 6 | 4 |

Assign each executive to one position each so that the sum of ratings for all the three positions is highest.

**Solution :** This being a maximization assignment problem, we first change it to a minimization assignment problem and to this end, we make a new matrix $[9 - c_{ij}]$, where 9 is the largest of all $c_{ij}$'s (one can take any number greater than or equal to the largest cost element) as follows :

| | | |
|---|---|---|
| 2 | 4 | 3 |
| 1 | 5 | 2 |
| 0 | 3 | 5 |

Obviously, the optimal assignment of the new matrix is also optimal for the original matrix. Now, to obtain the optimal assignment of the new matrix, we subtract 2, 1 and 0 from the $I^{st}$, $2^{nd}$ and $3^{rd}$ rows and then 0, 2 and 1 from the $I^{st}$, $2^{nd}$ and $3^{rd}$ columns to obtain the following :

| | | |
|---|---|---|
| 0 | 2 | 1 |
| 0 | 4 | 1 |
| 0 | 3 | 5 |

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 2 | 0 |
| 0 | 1 | 4 |

Clearly, the minimum number of horizontal and vertical lines required to cross-out all zeros is 3 which is equal to the order of the cost matrix. Hence, the optimal stage of assignment is reached.

We now put boxes round the single zeros in rows and columns and obtain the following :

| | I | II | III |
|---|---|---|---|
| $E_1$ | $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ |
| $E_2$ | $\cancel{0}$ | 2 | $\boxed{0}$ |
| $E_3$ | $\boxed{0}$ | 1 | 4 |

Hence, the optimal assignment is $E_1 \to$ II, $E_2 \to$ III and $E_3 \to$ I.

The highest sum of ratings = 5 + 7 + 9 = 21.

**Example 5.**Four operators are assigned to four machines with the effective costs given in the following table. Operator I cannot be assigned to Machine III and Operator 3 cannot be assigned to Machine IV. Find an optimal assignment for minimum cost in this case.

| | | Machines | | | |
|---|---|---|---|---|---|
| | | I | II | III | IV |
| Operators | 1 | 5 | 5 | – | 2 |
| | 2 | 7 | 4 | 2 | 3 |
| | 3 | 9 | 3 | 5 | – |
| | 4 | 7 | 2 | 6 | 7 |

**Solution :**Before applying the Hungarian method, we first note that the two assignments are impossible indicated by the dash (–) which shows the absence of any cost in the respective cells. So, if we assign costs greater than the largest of all $c_{ij}$'s, then automatically in the process, those will be pulled out. Hence, the starting cost matrix may be taken as :

| | I | II | III | IV |
|---|---|---|---|---|
| 1 | 5 | 5 | 10 | 2 |
| 2 | 7 | 4 | 2 | 3 |
| 3 | 9 | 3 | 5 | 10 |
| 4 | 7 | 2 | 6 | 7 |

Now, subtracting the smallest cost elements 2, 2, 3 and 2 respectively from the I$^{st}$, 2$^{nd}$, 3$^{rd}$ and 4$^{th}$ rows and then the smallest cost 3 from the I$^{st}$ column, we get

| | | | |
|---|---|---|---|
| 3 | 3 | 8 | 0 |
| 5 | 2 | 0 | 1 |
| 6 | 0 | 2 | 7 |
| 5 | 0 | 4 | 5 |

| | | | |
|---|---|---|---|
| 0 | 3 | 8 | 0 |
| 2 | 2 | 0 | 1 |
| 3 | 0 | 2 | 7 |
| 2 | 0 | 4 | 5 |

The minimum number of horizontal and vertical lines required to cross-out all the zeros is 3 which is less than the order 4 of the cost matrix. Hence, we subtract the smallest cost element 2 of the uncrossed cost element from the uncrossed cost elements and add 2 to the crossing cost elements to obtain the following :

| | | | |
|---|---|---|---|
| 0 | 5 | 8 | 0 |
| 2 | 4 | 0 | 1 |
| 1 | 0 | 0 | 5 |
| 0 | 0 | 2 | 3 |

Clearly, the minimum number of horizontal and vertical lines required to cross-out all the zeros is 4 now. So, an optimal stage of assignment is reached.

| | | | |
|---|---|---|---|
| 0 | 5 | 8 | 0 |
| 2 | 4 | 0 | 1 |
| 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 3 |

Now, drawing horizontal and vertical lines through the boxed zeros, we examine for singleness of zeros in the reduced matrix and obtain the cell (3, 2) looked horizontally and the cell (4, 1) looked vertically.

So, the complete assignment is given below :

| | | | |
|---|---|---|---|
| $\cancel{0}$ | 5 | 8 | $\boxed{0}$ |
| 2 | 4 | $\boxed{0}$ | 1 |
| 1 | $\boxed{0}$ | $\cancel{0}$ | 5 |
| $\boxed{0}$ | $\cancel{0}$ | 2 | 3 |

i.e. 1→IV, 2→III, 3→II and4→I.

∴ The minimum cost of assignment = Rs. (2 + 2 + 3 + 7) = Rs. 14.

**Note :** Note that Operators 1 and 3 have not got the restricted assignments.

**Example 6.**A company having four machines accepts three jobs, one to be assigned to one machine only. The cost of each job on each machine is given in the following table :

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | 18 | 24 | 28 | 32 |
| $J_2$ | 8 | 13 | 17 | 19 |
| $J_3$ | 10 | 15 | 19 | 22 |

Obtain an optimal assignment of the jobs to the machines.

**Solution :** We first note that in the given assignment problem, there are four machines and three jobs. Hence, to apply our method, we introduce a hypothetical job with zero cost at all the machines and thus get the starting cost matrix as :

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | 18 | 24 | 28 | 32 |
| $J_2$ | 8 | 13 | 17 | 19 |
| $J_3$ | 10 | 15 | 19 | 22 |
| $J_4$ | 0 | 0 | 0 | 0 |

Subtracting the smallest cost elements 18, 8, 10 and 0 respectively from the I$^{st}$, 2$^{nd}$, 3$^{rd}$ and 4$^{th}$ row, we get

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | 0 | 6 | 10 | 14 |
| $J_2$ | 0 | 5 | 9 | 11 |
| $J_3$ | 0 | 5 | 9 | 12 |
| $J_4$ | 0 | 0 | 0 | 0 |

Clearly, the minimum number of horizontal and vertical lines required to cross-out all the zeros is 2 which is less than the order of the cost matrix. Hence, we subtract the smallest cost element 5 of the uncrossed cost elements from all the uncrossed cost elements and add 5 to the crossing cost elements to obtain

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | 0 | 1 | 5 | 9 |
| $J_2$ | 0 | 0 | 4 | 6 |
| $J_3$ | 0 | 0 | 4 | 7 |
| $J_4$ | 5 | 0 | 0 | 0 |

Again, the minimum number of horizontal and vertical lines required to cross-out all the zeros is 3 which is less than order of the cost matrix. Hence, we subtract the smallest cost element 4 of the uncrossed cost elements from all the uncrossed cost elements and add 4 to the crossing cost elements to obtain

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | 0 | 1 | 1 | 5 |
| $J_2$ | 0 | 0 | 0 | 2 |
| $J_3$ | 0 | 0 | 0 | 3 |
| $J_4$ | 9 | 4 | 0 | 0 |

Now, the minimum number of horizontal and vertical lines to cross out all the zeros is 4 which equals to the order of the cost matrix. Hence, an optimal stage of assignment is reached. So, we start boxing the single zeros in the rows and columns and obtain

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | $\boxed{0}$ | 1 | 1 | 5 |
| $J_2$ | $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ | 2 |
| $J_3$ | $\cancel{0}$ | $\cancel{0}$ | $\boxed{0}$ | 3 |
| $J_4$ | 9 | 4 | $\cancel{0}$ | $\boxed{0}$ |

or

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| $J_1$ | $\boxed{0}$ | 1 | 1 | 5 |
| $J_2$ | $\cancel{0}$ | $\cancel{0}$ | $\boxed{0}$ | 2 |
| $J_3$ | $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ | 3 |
| $J_4$ | 9 | 4 | $\cancel{0}$ | $\boxed{0}$ |

Hence, the optimal solutions (assignments) are :

**(i)** $J_1 \rightarrow M_1$, $J_2 \rightarrow M_2$, $J_3 \rightarrow M_3$, with minimum cost = Rs. (18 + 13 + 19) = Rs. 50

**or (ii)** $J_1 \rightarrow M_1$, $J_2 \rightarrow M_3$, $J_3 \rightarrow M_2$, with minimum cost = Rs. (18 + 17 + 15) = Rs. 50

# MODEL OBJECTIVE QUESTIONS

**1.** An assignment problem is considered as a particular case of a transportation problem, because

**(a)** the number of rows equals the number of columns **(b)** all $x_{ij}$= 0 or 1.

**(c)** all rim conditions are 1. **(d)** all of the above.

**2.** An optimal assignment requires that the maximum number of lines which can be drawn through squares with zero opportunity cost be equal to the number of

**(a)**rows or columns. **(b)**rows and columns.

**(c)**rows + columns – 1. **(d)** none of the above.

**3.** While solving assignment problem, an activity is assigned to a resource through a square withzero opportunity cost because the objective is to

**(a)**minimize total cost of assignment

**(b)**reduce the cost of assignment to zero

**(c)**reduce the cost of that particular assignment to zero.

**(d)** all of the above.

**4.** The method used for solving an assignment problem is called.

**(a)**reduced matrix method. **(b)** MODI method

**(c)** Hungarian method. **(d)**none of the above.

**5.** The purpose of a hypothetical (i.e. a dummy) row or column in an assignment problem is to

**(a)**obtain balance between total activities and total resources.

**(b)**prevent a solution from becoming degenerate.

**(c)**provide the means of representing a dummy problem.

**(d)**non of the above.

**6.** Maximization assignment problem is transformed into a minimization problem by

**(a)**adding each entry in a column from the maximum value in that column.

**(b)**subtracting each entry in a column from the maximum value in that column.

**(c)**subtracting each entry in the table from the maximum value in that table.

**(d)**any one of the above.

**7.** If there were n workers and n jobs, then there would be

**(a)**$n$! solutions **(b)** ($n$-1) ! solutions
**(c)** ($n$!) $n$ solutions **(d)**$n$ solutions

**8.** An assignment problem can be solved by :

**(a)**simplex method **(b)** transportation method
**(c)**both (a) and (b) **(d)** none of the above.

**9.**For a salesman who has to visit n cities, which of the following is true for the number of ways of his tour plan ?

**(a)**$n$! **(b)** ($n$ +1)! **(c)** ($n$ – 1)! **(d)**$n$

**10.**The assignment problem :

**(a)**requires that only one activity be assigned to each resource.
**(b)**is a special type of transportation problem
**(c)**can be used to maximize resources
**(d)**all the above.

**11.**Which of the following should be preferred in making assignment
**(a)**a row with single zero**(b)** a column with multiple zeros
**(c)**a column with single zero **(d)** either (a) or (c).

**12.** In an assignment problem involving 7 workers and 7 jobs, the total number of assignment possible is equal to

**(a)** 7 **(b)** 14 **(c)** 49 **(d)**7 !

## ANSWERS

**1.**(d) **2.**(a)**3.** (a) **4.** (c) **5.** (a) **6.** (c)

**7.** (a)**8.** (c) **9.** (c) **10.** (d) **11.**(d) **12.** (d).