18. Quasi-neutral behaviour of plasma :-

Quasi-neutrality describes the apparent charge neutrality of a plasma overall, while at smaller scales, the positive and negative charges making up the plasma, may give rise to charge regions and electric fields.

Since electrons are very mobile, plasmas are excellent conductors of electricity, and any charges that develop are readily utilized, and in many, cases, plasmas can be treated as being electrically neutral. The term is sometimes written without the hyphen i.e. quasineutrality.

- For length scales larger than Debye length the charge separation is close to zero. One can thus use the approximation of quasi-neutrality -

$$\sum_i Z_i n_i = 0 , \quad n_i = n_e .$$

- Note that, this does not mean that there is no electric field in the plasma.

- Under the Quasi-neutrality approximation the Poisson's equation can no longer be used to calculate the electric field -

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \rightarrow \boxed{\rho = 0} .$$

- Plasma tends to be electrically neutral at each point due to charge-to-mass ratio ($e/m_e = 1.8 \times 10^{11}$ C/kg) of electrons.

- If charge neutrality breaks down at some point

⇒ Electric field is exerted around it.

⇒ Electrons are accelerated towards positive charge region.

⇒ Recovering a charge neutrality in a very short time.

19. Bohm-Gross Dispersion relation for Electron plasma wave :-

There is another effect that can can cause plasma oscillations to propagate, and that is thermal motion. Electrons streaming into adjacent layers of plasma with their thermal velocity will carry information about what is happening in the oscillation region. The plasma oscillation can then properly be called a plasma wave. We can easily treat this effect by adding a term $-\nabla p_e$ to the eqn of motion

$$mn_e\left[\frac{\partial v_e}{\partial t} + (v_e\cdot\nabla)v_e\right] = -en_eE \quad \cdots\cdots(1)$$

and for isothermal compression, we have-

$$\nabla p = \nabla(nkT) = KT\nabla n.$$

So that, clearly $\gamma = 1$. For adiabatic compression, KT will also change, giving γ a value greater than 1. If N is the number of degrees of freedom, γ is given by -

$$\gamma = \frac{(2+N)}{N} \quad \cdots\cdots(2).$$

Hence,

$$\nabla p_e = 3k_e T_e \nabla n_e$$
$$= 3kT_e \nabla (n_0 + n_1) = 3KT_e \frac{\partial n_1}{\partial x} \hat{x}.$$

and the linearization eqn of motion is –

$$mn_0 \frac{\partial v_1}{\partial t} = -en_0 E_1 - 3KT_e \frac{\partial n_1}{\partial x} \quad \ldots.(3)$$

Note that in linearizing we have neglected the terms $n_1 \frac{\partial v_1}{\partial t}$ and $n_1 E_1$ as well as the $(v_1 \cdot \nabla) v_1$ term.

The oscillating quantities are assumed to behave sinusoidally:

$$\left.\begin{aligned} v_1 &= v_1 e^{i(kx - \omega t)} \hat{x} \\ n_1 &= n_1 e^{i(kx - \omega t)} \\ E &= E e^{i(kx - \omega t)} \hat{x} \end{aligned}\right\} \quad \ldots\ldots(4)$$

With eqn (4), eqn (2) can be written as –

$$im\omega n_0 v_1 = \left[en_0 \left(\frac{-e}{ik\epsilon_0} \right) + 3kT_e ik \right] \frac{n_0 ik}{i\omega} v_1 \quad \ldots(5)$$

$$\left\{ \begin{aligned} &\text{After using} \\ &E_1 = \frac{-en_1}{ik\epsilon_0} \text{ and} \\ &n_1 = \frac{n_0 ik v_1}{i\omega} \end{aligned} \right\}$$

or, $$\omega^2 v_1 = \left(\frac{n_0 e^2}{\epsilon_0 m} + \frac{3KT_e}{m} k^2 \right) v_1 \quad \cdots(6).$$

or, $$\boxed{\omega^2 = \omega_p^2 + \frac{3}{2} k^2 v_{th}^2} \quad \ldots(7) \quad \left\{ \begin{aligned} &\text{Here, } \omega_p = \frac{n_0 e^2}{\epsilon_0 m}, \\ &\& \ v_{th}^2 = \frac{2KT_e}{m}. \end{aligned} \right\}$$

Where, $v_{th}^2 = \frac{2KT_e}{m}$. The frequency now depends on k, and the group velocity is finite, given by:

$$2\omega\, d\omega = \frac{3}{2} v_{th}^2\, 2k\, dk.$$

$$v_g = \frac{d\omega}{dk} = \frac{3}{2}\frac{k}{\omega} v_{th}^2$$

or $\boxed{v_g = \frac{3}{2}\frac{v_{th}^2}{v_\phi}}$ .....(8). $\left\{ \text{Here } v_\phi = \frac{\omega}{K} \text{ phase velocity} \right\}$

That v_g is always less than speed of light 'c', can easily be seen from a graph of eq^n (7).

Equation (7) is the ~~dis~~ Bohm-Gross dispersion relation $\omega(k)$.

The plot of the dispersion relation $\omega(k)$ as given by equation (7) is shown below-

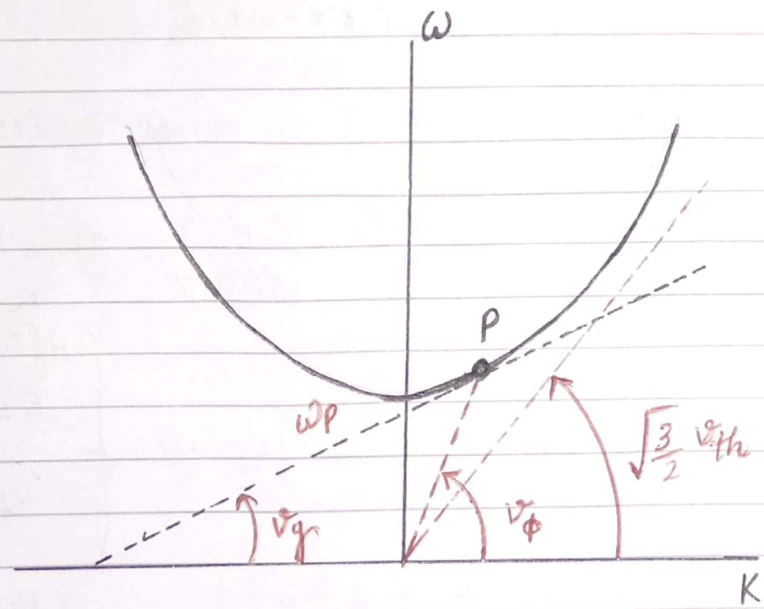

Fig.(a): Dispersion relation for electron plasma waves (Bohm-Gross waves).

At any point P on this curve, the slope of a line drawn from the origin gives the phase velocity ω/K. The slope of the curve at P gives the group velocity (V_g). This is clearly always less than $(3/2)^{1/2} v_{th}$, which, in our nonrelativistic theory, is much less than "c". Note that at large k (small λ), information travels essentially at the thermal velocity. At small k (large λ), information travels more slowly than v_{th} even though v_ϕ is greater than v_{th}. This is because the density gradient is small at large λ, and thermal motions carry very little net momentum into adjacent layers.

The existance of plasma oscillations has been known since the days of Langmuir in the 1920s. It was not untill 1949 that Bohm-Gross worked out a detailed theory telling how the waves would propagate.

20. The invariance of 'J' for plasma in a static nonuniform magnetic field :-

A particle bouncing between turning points a and b in a magnetic field is shown below -

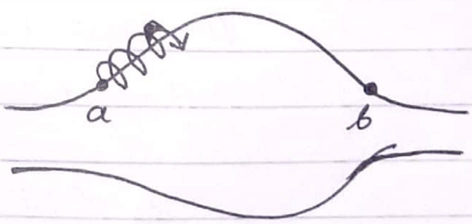

Fig. (a).

Proof of the invariance of J :-

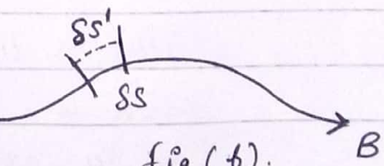

fig (b).

To prove the invariance of J, we first consider the invariance of $V_{\parallel} \delta s$, where δs is the segment of the path along B as shown in fig (b).

Because of the guiding centre drifts, a particle on s will find itself on another line of force $\delta s'$ after a time Δt. The length of ss' is defined by passing planes perpendicular to B through the end points of δs. The length of δs is obviously proportional to the radius of curvature:

$$\frac{\delta s}{R_c} = \frac{\delta s'}{R_c'}$$

so that,

$$\frac{\delta s' - \delta s}{\Delta t \cdot \delta s} = \frac{R_c' - R_c}{\Delta t\, R_c} \quad \ldots\ldots (1)$$

The "radial" component of V_{gc} is just

$$V_{gc} \cdot \frac{R_c}{R_c} = \frac{R_c' - R_c}{\Delta t} \quad \ldots\ldots (2)$$

We know that,

$$V_{gc} = V_{\nabla B} + V_R = \pm \frac{1}{2} v_\perp r_L \cdot \frac{B \times \nabla B}{B^2} + \frac{m v_{\parallel}^2}{B^2} \frac{R_c \times B}{R_c^2 B^2} \quad \ldots\ldots (3)$$

The last term has no component along R_c. Using eq^{n}s (2) and (3), we can write equation (1) as —

$$\frac{1}{\delta s}\frac{d\,\delta s}{dt} = V_{gc} \cdot \frac{R_c}{R_c^2} = \frac{1}{2}\frac{m v_\perp^2}{q B^3}(B \times \nabla B) \cdot \frac{R_c}{R_c^2} \quad \ldots (4)$$

This is the rate of change of δs as seen by particle. We must have the rate of change of $V_{\parallel}$ as seen by the particle. The parallel and perpendicular energies are defined by —

$$W \equiv \frac{1}{2} m v_{\parallel}^2 + \frac{1}{2} m v_{\perp}^2 = \frac{1}{2} m v_{\parallel}^2 + \mu B = W_{\parallel} + W_{\perp} \quad \cdots\cdots (5).$$

Thus $V_{\parallel}$ can be written,

$$V_{\parallel} = \left[(2/m)(W - \mu B)\right]^{1/2} \quad \cdots\cdots (6)$$

Here W and μ are constant, and only B varies. Therefore -

$$\frac{\dot{v}_{\parallel}}{v_{\parallel}} = \frac{-1}{2} \frac{\mu \dot{B}}{W - \mu B} = \frac{-1}{2} \frac{\mu \dot{B}}{W_{\parallel}} = \frac{-\mu \dot{B}}{m v_{\parallel}^2} \quad \cdots\cdots (7).$$

Since B was assumed static, $\dot{B}$ is not zero only because of the guiding centre motion:

$$\dot{B} = \frac{dB}{dr} \cdot \frac{dr}{dt} = V_{gc} \cdot \nabla B = \frac{m V_{\parallel}^2}{q} \frac{R_c \times B}{R_c^2 B^2} \cdot \nabla B \quad \cdots\cdots (8).$$

Now we have,

$$\frac{\dot{V}_{\parallel}}{V_{\parallel}} = -\frac{\mu}{q} \frac{(R_c \times B) \cdot \nabla B}{R_c^2 B^2} = -\frac{1}{2} \frac{m}{q} \frac{V_{\perp}^2}{B} \frac{(B \times \nabla B) \cdot R_c}{R_c^2 B^2} \quad \cdots\cdots (9).$$

The fractional change in $V_{\parallel} \delta s$ is-

$$\frac{1}{V_{\parallel} \delta s} \frac{d}{dt}(V_{\parallel} \delta s) = \frac{1}{\delta s} \frac{d \delta s}{dt} + \frac{1}{V_{\parallel}} \frac{d V_{\parallel}}{dt} \quad \cdots\cdots (10).$$

From eqⁿs (4) and (9), we see that these two terms cancel, so that,

$$\boxed{V_{\parallel} \delta s = \text{constant}} \quad \cdots\cdots (11)$$

This is not exactly the same as saying J is constant, however, In taking the integral of $V_{\parallel} \delta s$ between the turning points, it may be that the turning points on SS' do not coincide with intersections of the perpendicular planes (fig b). However, any error in J arising from such a discrepancy is negligible because near the turning points, $V_{\parallel}$ is nearly zero. Consequently, we have proved -

$$\boxed{J = \int_a^b V_{\parallel} \, ds = \text{constant}} \quad \ldots\ldots (11)$$

Proved.